Be a cause of the future and not a result from the past. Futures expectations in financial markets are paramount for today’s decision-making processes. Although many indicators have been developed in recent years to gauge investors’ expectations, most only give a narrow view of the situation, signalling just bullish or bearish sentiment. In this report, we go beyond those measures and aim to identify the exact probabilities players attach to each possible future price of a specific asset. Hence, commanding this set of probabilities, we should be in a better position to control present investment decisions.

Options markets are full of expectations. In derivatives, the future influences the present much more than the past. It is well known that options give the right to buy or sell an asset at a point in the future at a price set now (the strike). If we look at options to buy an asset at a particular point in the future but at different strike prices, the prices at which such contracts trade tell us something about the investors’ view of the probabilities that the price of the underlying will be above the various strike prices.

Extracting implied expectations from options prices. This report explores how to extract these probabilities from options prices. We calculate the likelihood the options market attaches to different price levels of the underlying instrument at the end of the term of the contract (June 08, Sep 08 and Dec 08). Technically speaking, we obtain the risk-neutral probabilities from options prices following a methodology used by some Central Banks to make monetary policy decisions.

What’s priced in? The case of the Spanish market. The assets considered are the IBEX35 index and seven companies representing over 70% of total Spanish companies’ market cap. The results show that the most likely scenario is that the IBEX35 will finish the year at 14900. However, large uncertainties remain and the probabilities are clearly biased to a negative scenario. On top of that, the probability attached to the lower tail of the distribution is high, indicating that the markets believe that we could experience the ‘unlikely’ event of a crash.
# CONTENTS

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>EQUITIES ARE FROM MARS, DERIVATIVES FROM VENUS</strong></td>
<td>3</td>
</tr>
<tr>
<td><strong>INTRODUCTION</strong></td>
<td>3</td>
</tr>
<tr>
<td><strong>BACK FROM THE FUTURE</strong></td>
<td>4</td>
</tr>
<tr>
<td><strong>A RISK NEUTRAL WORLD</strong></td>
<td>5</td>
</tr>
<tr>
<td><strong>THE NATURE OF OPTIONS-IMPLIED EXPECTATIONS</strong></td>
<td>6</td>
</tr>
<tr>
<td><strong>WHAT’S PRICED IN? THE CASE OF THE IBEX35</strong></td>
<td>7</td>
</tr>
<tr>
<td><strong>CONCLUSIONS: BACK TO THE PRESENT</strong></td>
<td>11</td>
</tr>
<tr>
<td><strong>TECHNICAL APPENDIX</strong></td>
<td>12</td>
</tr>
<tr>
<td>Extracting Probability Distribution Functions</td>
<td>12</td>
</tr>
<tr>
<td>What’s Priced In? A Look at Some Spanish Stocks</td>
<td>14</td>
</tr>
<tr>
<td>Who is Who in RNDs</td>
<td>17</td>
</tr>
<tr>
<td><strong>BIBLIOGRAPHY</strong></td>
<td>18</td>
</tr>
</tbody>
</table>
EQUITIES ARE FROM MARS, DERIVATIVES FROM VENUS

‘Successful investing is anticipating the anticipations of others.’ John Maynard Keynes, Economist.

‘The quality of expectations determines the quality of our action.’ A Godin, French writer.

INTRODUCTION

In general, we should all be concerned about the future as we will have to live (hopefully) our lives there. In financial markets, the future takes another dimension. Hence, expectations about what can happen over the investing period are a key variable for today’s decision making processes (portfolio selection, risk management, conduct of monetary policy decisions, etc). Contrarians like to bet against the general mainstream, so any insight about what consensus expects is very valuable to them. On the other hand, followers of the trend would be pleased to know where consensus expects the markets will be heading over the next months. This interest in knowing what other market participants expect for the future has led to different measures to gauge investors’ expectations. These indicators have been developed extensively in recent years (e.g., VIX, Put/Call ratios, commitment of traders reports, information from volumes) to help markets participants make decisions.

However, all these indicators fall short of giving complete information. Most of the time, at best they reveal no more than the general direction of the market (bearish or bullish sentiment), while at others, they just reflect an indirect measure of sentiment (panic or euphoria).

In this report we try to go beyond these somewhat simplistic measures. Our goal is to extract a full set of information from the markets. We aim to identify the specific probability that market participants attach to different asset values at a pre-determined future date. And, where is the best place to find this information? Well, we believe it will be where market participants make their investments, as these should be a reflection of their sentiment. This place is commonly known as the options market.

In the past decade some methodologies were developed with the purpose of extracting market expectations implicit in options prices. Basically, the idea consisted of recovering the density functions (probability distribution function, pdfs) of the price of the underlying asset on the maturity date of options negotiated on the market. Institutions like the BoE or the IMF are among those that use the information in the form of density functions extracted from options to take monetary policy decisions or contrast it with their own forecasts.

In this report, we explain our approach to the problem, examining the concept behind the methodology (see appendix for technical details). Finally, we will also apply the model to the Spanish market, analysing investors’ expectations for the IBEX35, Telefónica, Santander, BBVA, Banco Popular, Repsol, Iberdrola, Gas Natural and Inditex for the coming quarters (June 2008, September 2008 and December 2008).
BACK FROM THE FUTURE

Today’s derivative markets provide investors with a rich source of information for gauging market sentiment. Due to their forward-looking nature, options prices efficiently encapsulate market perceptions about underlying asset prices in the future. Options prices reveal genuinely interesting information about the expected path for the underlying asset.

Options prices include embedded expectations for futures asset prices. One way to see this, for example, is to calculate the variance that is implied by an option’s price. This value is the market’s ex ante estimate of the underlying asset’s return volatility over the remaining life of the option. The volatility implicit in option prices can be used to gauge the probabilities that market participants assign to different price levels of the underlying instrument over the term of the option. However, volatility is a narrow measure of uncertainty because it describes only the width or dispersion of the implied Probability Distribution Function (pdf) or, in plain words, volatility just measures the dispersion of the function that describes the range of future values the asset can take.

More interesting, however, would be to derive the higher moments of future asset values from the market prices of equity options. So, for example, we can calculate the third (skew) and the fourth (kurtosis) moment of the asset price distribution. The implied skew measures the asymmetry in the expectations of the operators in the options market around the expected value. The kurtosis indicates the market’s view of the likelihood of extreme price changes either way.

Specifically, we are be able to obtain where markets participants expect prices for a particular asset will be in the future using equity option market quotes. In this report, we travel into the future (using the options markets as a sort of time machine), take a snapshot of investors’ expectations, and return to the present with that information.

How exactly can we do that? The common method to value an option contract on a specific underlying assumes, among other things, a probability distribution function for the asset (also known as stochastic process, a fancy word derived from the Greek stochos or ‘random’). In Figure 1, we can see the Black-Scholes model’s assumed probability distribution and the hypothesised volatility term structure (B-S assumes constant volatility or a ‘flat smile’) to value options for the IBEX35.

Obviously, the current quoted option prices depend on the price of the underlying at maturity. So, options prices depend on the investors’ expected probability distribution of the underlying asset. In this report, we will work reversing the option valuation process. Looking at the quoted option prices (or implied volatilities as these are the mirror image of prices) we will obtain which is the implicit probability distribution for the underlying (see Figure 2). This density function will show us the probability market participants are giving to each value the asset can take at the option contract’s maturity.
From a technical point of view, the information we want can be extracted in the form of an *ex ante* risk-neutral probability distribution of the underlying price at the option’s maturity date.

*(Reader: Excuse me, hold on a second and let me breathe... An ex-ante risk-neutral probability distribution? Could you please elaborate this concept?)*

**A RISK NEUTRAL WORLD**

Einstein once said: ‘*Make everything as simple as possible, but not simpler*. Risk neutral probabilities are the key concept to understanding our approach to glean future expectations from the options markets. Following Einstein’s advice, we aim to explain this concept using simple concepts, but without making the concept simpler than necessary (see the appendix for those more mathematical skilled).

If this were a script from the *Twilight Zone* TV series, the risk-neutral world would exist in a parallel dimension. We could imagine the risk-neutral world as one identical to the one we current live in. The only difference would be that *people who live in the risk-neutral world are only risk-neutral*, while those in our world are either risk-averse or risk-loving.

Let’s say that a citizen wants to insurance his car. Suppose he has a 1% chance of losing €1,000 from a car accident. The expected loss of this is €10, meaning that if he takes the car out 100 times, he will have an accident which would cost him €1,000 on one journey. The €1,000 total repair bill, when averaged over 100 trips, is €10 per trip.

In the risk-neutral world, on any given trip the citizen does not care if he pays €10 to the insurance company or is stuck with the 1% chance of a €1,000 loss. In our world, risk-averse people are willing to pay more than €10 and risk-loving people are willing to pay less.
Now our citizen wants to make an investment. Investment works much like insurance, but in the opposite direction. Suppose that our citizen has a 1% chance of winning, rather than losing, €1000 buying some biotechnology stock. What price would he be willing to pay for such an opportunity? The expected gain from the investment is €10, meaning that buying this company 100 times would have led to one win of €1000. The €1000 total winnings would then average out to €10 for the 100 investments.

Much like insurance, a risk-neutral investor is an in-between sort of person who would feel the same about paying €10 for the 1% chance of winning €1000 or keeping the €10. A risk-averse person, however, would be willing to pay less than €10 for this chance of winning as she/he prefers to have certainty (a sure €10) over uncertainty (the chance of winning €1,000). And a risk lover would be quite eager to pay more than €10, because he would enjoy the adrenaline rush of a risky situation.

All sorts of people, risk-neutral, risk-averse, and risk-loving, are willing to take the investment or the insurance if the price is right. As a risk-averse person, there’s no contradiction if I am willingly to pay €15 to avoid the possibility of a 1% chance of losing €1,000, while at the same time pay €5 for a 1% chance of winning €1,000.

THE NATURE OF OPTIONS-IMPLIED EXPECTATIONS

Those who already have clearly in mind what the mechanics of options are and the associated risk-neutral probabilities may profitably skip this section and go to the section on What’s Priced In? The Case of the IBEX35 on page 7. All others, by carefully studying what follows, should be able to glean a certain amount of confusion from it.

Derivatives are priced in a risk-neutral world. In an options contract, both the derivative and its underlying asset are subject to the same source of price changes or, equivalently, the same sources of risk. Thus, if the underlying asset is traded in the market, it may be possible to use the underlying to replicate a derivative, ie, to produce the same payoffs as the derivative asset. It can be shown that this can be achieved by combining, in the right proportions, the underlying asset with a risk-free asset (eg, a government bond). So, there is a no-arbitrage relationship between the price of the derivative and the value or price of the constructed portfolio (the two must be equal for there to be no opportunity for market participants to earn a riskless profit). In this sense, the derivative has a unique price that does not depend on the risk preferences of investors. So, the price of the derivative should be the same whether the derivative is valued by risk-neutral or risk-averse market participants.

This line of reasoning greatly simplifies the task of pricing derivative assets. Derivatives assets, like all financial assets, are priced by evaluating the expected future payoff from holding the asset over the term of the option. The expected future payoff must then be discounted to the present to express it in current prices. Valuing a derivative in this way requires a view on the expected rate of return of the asset. In a risk-neutral world, the expected rate of return for all assets is the risk-free interest rate, which is known, or at least can be easily approximated.

And where does this take us? Valuing derivatives in this fashion lead us to the following point: the price of European call options of a given maturity, but with a range of different exercise prices, are related to the weights attached by the representative risk-neutral agent to the possible outcomes for the price of the underlying security at the option’s maturity.
What is the effect of assuming risk neutrality in a risk-averse world when working in reverse? Although the price of derivatives should be the same in a risk-neutral or risk-averse world, the interpretation to be put on information inferred from derivative prices may well differ in the two cases. Our pdfs are extracted from option prices by using the risk-free rate of interest as the expected rate of return. As such, the information we obtain from the pdf reflects market expectations in a risk-neutral world. However, it is generally accepted that investors are risk-averse and will potentially have different expectations to those in a risk-neutral world. The most obvious effect on implied pdfs of a change from a risk-neutral world to a risk-averse world is on the mean of an implied pdf. The mean of an implied pdf is equal to the futures price of the underlying asset. Futures prices are risk-neutral prices and have been shown to be biased expectations of actual future spot prices. The difference is attributed to a risk premium. This premium is necessary to compensate risk-averse investors for the riskiness of the underlying asset. So, one of the implications of extracting implied pdfs from options prices by assuming investors are risk-neutral is a lower mean than in the case of a risk-averse world. Turning to the higher moments of implied pdfs (dispersion, asymmetry and kurtosis), academic work thus far suggests that, outside of periods of extreme market turbulence, the assumption of risk neutrality has little impact. Overall, this means that the risk-neutral assumption seems to be more important for the location of an implied pdf than it is for the shape of the distribution. As a result, the risk-neutral probabilities that we extract for different levels of the underlying asset in the future will be higher than those actually held by market participants.

Why should investors care about RND (risk-neutral density)? Investors should have an obvious interest in RND, since it gives a view on the market’s expectations. We can compare the RND with our subjective probability assessment of the future market outcome, and act accordingly. Also, analysis of RND changes could be a useful tool to evaluate typical market reactions to different extreme events.

Over the last few years, there has been considerable interest among academics, market participants and policy-makers in extracting information of this kind from options prices. For example, the Monetary Policy Committee of the Bank of England uses information from options prices on sterling interest rates to assess the degree of market uncertainty. Also, pdfs have proved useful in estimating the market’s assessment of the balance of risks associated with future movements in asset prices.

**WHAT’S PRICED IN? THE CASE OF THE IBEX35**

What are the market expectations (risk-neutral probabilities) implicit in IBEX35 options prices? If we were asked where the IBEX35 could be in a year from now, one could say, in theory at least, that it could take any value from zero to infinity. Common sense, however, tells us that this range is too big and the outcomes that are viewed as likely will form a small subset of it. Nevertheless, the broad range that prices could take illustrates an important point, ie, when considering the probability of an asset price being a specific value in the future, that specific price is just one of a possible infinite number of values. This is highlighted when looking at how probabilities attached to different asset price levels vary or are ‘distributed’ over alternative price levels (the probability distribution function, pdf).

---

1 Rubinstein (1994) converts an RND function for an equity index to a ‘consensus subjective’ density function under the assumption that the investor maximises his expected utility of wealth with constant relative risk aversion. He finds that for assumed market risk premia of between 3.3%-5%, the subjective distribution is only slightly shifted to the right relative to the risk-neutral distribution, and that the qualitative shapes of the two distributions are quite similar.
Figure 3 shows the pdf for the IBEX35 on April 14, 2008 extracted from quoted options prices. The x-axis shows future levels of the IBEX35 and the y-axis the probabilities of these levels occurring. It is clear from the magnitudes of the probabilities on the y-axis that the probability of any individual level occurring in the next three quarters is very low. Because of the small probability attached to one particular level occurring, it is often more useful to look at probabilities attached to the asset price lying in a particular range.

We can also look at the probability that the asset price will be at most at a particular level, i.e., the **probability that the asset price level will be less than or equal to a specified price**. This can be calculated from the pdf by summing up all of the area under the pdf curve up to the specified price level. Figures 3 and 4 illustrate this. For example, we can see how the probability that the IBEX35 will be at most at the current 13200 points (vertical red line) in December is given by area A (or 35% probability). The total area under the pdf curve must add up to one. See the appendix for the outcome in Telefónica, BBVA, Banco Santander, Repsol, Iberdrola, Gas Natural, Inditex and Banco Popular.

---

**Figure 3. Where Do Investors Expect the IBEX35 to be in the Quarters Ahead?**

![Graph showing the expected levels of the IBEX35 and the probabilities associated with each level.](image)

*Note: The graph represents the implied pdf for the IBEX35 extracted using the mixture of two log-normals (see appendix). The x-axis represents the range of values in the IBEX35 and the y-axis represents the probability attached to each of the values by the options market. The red vertical line represents current prices and the black represents the mode of the distribution (the most likely scenario). Source: MEFF and Santander Investment Bolsa.*

**Figure 4. Accumulated Probability for a Specific Value of the IBEX35 in the Quarters Ahead**

![Graph showing the accumulated probability for the IBEX35.](image)

*Note: The graph represents the accumulated probability for the IBEX35 extracted using the mixture of two log-normals (see appendix). The x-axis represents the range of values in the IBEX35 and the y-axis represents the accumulated probability attached to each of the values by the options market. For example, the options market assigns a 35% probability that the IBEX35 will be lower than current prices at year-end, as represented by the red vertical line. Also, 14200, 14300 and 14700 points are the levels that would leave a 50% probability to each side of the distribution for each maturity date, as represented by the black vertical lines. Source: MEFF and Santander Investment Bolsa.*

---

2 In order to avoid the problems associated with asynchronous intra-day quotes we use MEFF’s settlement prices established at the end of each day. These prices are used as the basis for overnight ‘marking-to-market’ of all open positions. Hence, they should give a fair reflection of the market at the close of business, at least for contracts with open interest.
In the particular case of the IBEX35, the analysis of the distribution obtained shows us that the market believes that, most likely, the IBEX35 could advance over the next quarters. We can see how the mode of the distribution (the most likely scenario) is placed further to the right (higher levels for the IBEX35) as the quarters pass. Specifically, the options market believes that the most probable scenarios are that the IBEX35 could finish June at 14100 points, September at 14600 and December at 14900 points (see the vertical black lines in Figure 3). However, in the December maturity, the distribution is almost bi-modal, suggesting that investors are not ruling out a possible crash scenario, with the index priced at 11500 points.

As stated earlier, another well used probability distribution is that of the cumulative distribution function (cdf). The information in this distribution is complementary to that in the pdf. Figure 4 shows the cdf corresponding to the pdf shown in Figure 3. The y-axis now shows probabilities that the asset price will be less than or equal to those levels specified on the x-axis. Continuing with the IBEX35 example above, the y-axis value in the cdf corresponds to area A under the pdf in Figure 3. We can see how the market is discounting a 35% probability that the IBEX35 will finish the year below current levels. On the Sep 08 contract, investors attach a 35% probability that the index could be below the current level, and on Jun 08 the probability amounts to 20%. The increase in this accumulated probability, from 20% in the June maturity to 35% in September and 35% in December, suggest that the market attaches a higher probability to a scenario where the index could be lower at the September and December maturities than in June.

However, we have seen that the most likely scenario will be for the index to advance to 14600 points in September and 14900 points in December. Is there an inconsistency here? Not really, the answer has to do with the difference between the mean and mode.

A simple example will illustrate this difference. Let’s imagine that we will be paid €100 if the IBEX35 falls below 12000 points in Jun 08, an event with a 10% probability. If, on the other hand, the IBEX35 is above the 12000 level, we will be paid nothing (90% probability). Well, the expected value (the mean) of this game is €10 (10%*€100+90%*0), while the most likely scenario (mode) is that we will be paid nothing (a 90% probability that the index will finish the year above 12000).

In the extracted pdf in the IBEX35, there is a strong increase in the skew of the distribution from June to December. The skew is biased to the negative side, indicating a higher probability that stocks could drop in value. This negative skew drags down the expected value of the distribution, increasing the accumulated probability that the index could be below current levels.

We can also look at the median or the level of the index that leaves a 50% chance that it will be above and below. According to the cdf, there is a 50% chance that the IBEX35 will finish above or below the 14200 points level in June, 14300 in September and 14700 in December.

How is market uncertainty reflected in the pdfs? Changes in the width or dispersion of the distribution can inform us about changes in market uncertainty about future asset price levels.

Figure 3 compares the three, six and nine month implied pdf for the IBEX35 on April 14, 2008. The dispersion of the pdf increases the longer the maturity. This suggests that markets are relatively more uncertain about future IBEX35 prices over the subsequent nine months than over the next three months. The standard deviation of the implied pdf and/or the option implied volatility are commonly used statistics to measure this dispersion or market uncertainty.
The 'Time to Maturity' Effect. Options contracts traded on exchanges have fixed expiry dates. Each day, the implied pdfs that are extracted from these contracts tell us something about the market’s views on the possible change in the underlying asset price between that day and the expiry date. But, day by day, the horizon gets closer. This matters when we compare movements over time in the shape of the implied pdfs. The degree of uncertainty about the price of the underlying futures contract at the expiry date of the option naturally decreases as the expiry date approaches. So, our estimated variances of the pdfs diminish over time, without any real change in the degree of uncertainty about the asset.

In the absence of any unexpected shocks, one would expect volatility to decline the closer we get to the expiry date. Even if the market’s degree of uncertainty is unchanged, the dispersion data obtained from fixed-expiry contracts would be expected to show a decline. This is a weakness in the interpretation of changes in RNDs as time passes. How much of this can be attributed to the effect of new information in the market and how much is merely an effect of less time to maturity? On a day to day basis, and if there is a long time to maturity, the time effect is insignificant. But over longer periods of time, a month for example, the effect can affect the interpretation.

Although this is not the aim of this report, if we wanted to compare RND changes over time and isolate the time effect, we could interpolate between two volatility smiles, these being estimated at the same date but from options with different expiry dates. In this fashion, we obtain a constant maturity smile and are able to extract constant horizon RND following the Shimko (1993) non parametric approach (see appendix for more information regarding RND calculation methods). Corrected for this effect, changes over time in RND can be attributed to a large extent to changes in market expectations.

In upcoming reports we plan to calculate how constant horizon RND in the IBEX35 has changed in recent months, allowing us to see how investors’ expectations have varied over time. However, as we can see in the appendix we will need to use a non parametric model to extract RNDs stripped of the ‘time effect’.

What is the market’s balance of risk? The degree of asymmetry of distribution can tell us something about the market’s assessment of the relative risks of future asset price moves in one direction relative to the other. Figure 3 illustrates the asymmetry of the IBEX35 pdfs. The pdf suggests that the market attached very different probabilities to outcomes above and below the mode. In the three periods considered the options markets puts more emphasis on levels below the mode than on those above it. This negative asymmetry suggests that the market’s assessment of the ‘balance of risks’ over the rest of the year points to expectations of lower, rather than higher returns over the subsequent nine months. The skew of the implied pdf is a commonly used statistic to measure the degree of asymmetry.

And what about ‘Black Swans’? In ‘The Black Swan: The Impact of the Highly Improbable’, Nassim Nicholas describes a ‘Black Swan’ as an extremely rare event (black swans were once thought not to exist until they were found in Australia in the 17th century). On August 19, 1991 IBEX35 saw its biggest fall in history, posting a one-day decline of 9.01%. Following the standard paradigm, let’s assume that IBEX35 prices are log-normally distributed with a calculated daily volatility of 1.1% and a 0.05% mean (calculated over daily returns from 1993-2008). Under the log-normal hypothesis, this is a -9 standard deviation event, which has a probability of $2.42 \times 10^{-34}$ and should occur only once in $1.63 \times 10^{11}$ years. Even if we had lived through the entire 13bn-year life of the universe and lived this life 13bn times, such a decline (9.01%) would have occurred just once in this period. Expect the unexpected.
Interestingly, it is now well known that since the 1987 crash, Black-Scholes-implied volatilities for S&P 500 index options have consistently exhibited pronounced smile effects.

Hence, the **probabilities attached to outcomes that are far** from current asset prices, or the degree of ‘fatness’ of the tails of the pdf, is certainly interesting. It can help us to assess the market’s expectations for the likelihood for extreme changes in future asset prices.

The **kurtosis** (indicates how fat the tails are) **measures the probability the market attaches to extreme levels**, either up or down. Levels of kurtosis above three indicate that the market attaches higher probability to extreme outcomes than would be implied by a normal distribution.

In the **IBEX35** case (and also in most of the companies analysed in the appendix), we can see the same pattern: the **probability density in the tails of the pdf**, ie, those regions far from the most likely scenario (reflected in the centre of the pdf) **is much higher to the left** of the distribution than to the right. These heavier tails on the left are consistent with a market perception of a **greater chance of a large and ‘unlikely’ drop in equity prices** than to a large increase.

**CONCLUSIONS: BACK TO THE PRESENT**

As we can see, extracting risk-neutral densities from options markets is a complex task. However, we believe that the effort is worthwhile. **We can see that it is possible to extract information from equity options regarding the expected trajectory of asset prices.** Moreover, the implicit expectations extracted from the derivatives markets yield more information than common implicit measures (implied volatility, put/call ratios, etc.).

We have **extracted all the risk-neutral expected density** for the IBEX35 and the eight biggest companies in the Spanish market (accounting for over 70% of the index) which provide us with information on the expectations of economic agents regarding the distribution of the probability of these underlying assets on the date of option maturity.

However, interpretation of these risk-neutral densities is no easy task and the use of these densities in helping to make investment decisions is controversial. Do the RNDs implied in the price of options contracts on the Spanish IBEX35 index accurately predict the distribution of future outcomes of the index? We will try to answer this in subsequent reports. However, **the message is clear: these densities possess valuable information that must be taken into consideration in constructing relevant scenarios for the future.**

In the particular **case of the IBEX35**, the market seems to consider that the **most likely scenario is the IBEX35 advancing in the next quarters**. On the Jun 08 contract, the options market indicates that the most likely scenario is that the IBEX35 could stand at around 14100 points. For Sep 08, the IBEX35 is expected to advance to 14600 and to 14900 for Dec 08. **However, a large amount of uncertainty remains.** There is a **strong asymmetry** in the distribution; the pdfs are clearly biased to the left, indicating that investors still assign a higher likelihood to a negative scenario than to a positive one. On top of that, **the probability attached to the lower tail of the distribution is very high**, not only in absolute terms but also compared with the higher tail. **The markets are sticking to the scenario that they could experience the ‘unlikely’ event of a market crash.**
TECHNICAL APPENDIX

EXTRACTING PROBABILITY DISTRIBUTION FUNCTIONS

There are different methodologies available to estimate asset pdfs from a range of derivatives instruments for that particular asset. In particular, there exist two basic approximations to the problem: (a) Parametric. In this technique we assume a specific parametric form for the pdf, and that the underlying asset follows a particular probability distribution (log-normal, mixture of two log-normals). The parameters of the pdf are estimated in such a way that the implied call price function is as close as possible to that actually observed in the data. The mixture of two log-normals is the distribution we assume in this work to obtain the underlying pdfs; (b) Non parametric. Using quoted options prices we build the probability distribution for the asset without assuming a pre-defined structure.

The Parametric Method

The parametric estimation approach involves specifying a particular functional form for the pdf, \( f(S_T) \), and fitting this distribution to the observed range of strike prices via non-linear least squares. Although a range of forms has been suggested, the most commonly used is the mixture of two log-normal distributions. This is the functional form we assume in our report.

This form is sufficiently flexible to capture features that we might expect to find in the data, such as fatness in the tails of the distribution (excess kurtosis), positive or negative skew, or bimodality (see Melick and Thomas, 1997). Also, the mixture of two log-normals method is parsimonious, in the sense that it can be derived by estimating only a few (five) parameters.

The mixture of two log-normals \( f(S_T) \) is given by:

\[
 f(S_T) = \theta L(\alpha_1, \beta_1) + (1-\theta) L(\alpha_2, \beta_2) \tag{1}
\]

\[
 \alpha_i = \ln(S_T) + (\mu_i - 0.5\sigma_i^2)\tau
\]

\[
 \beta_i = \sigma_i \sqrt{\tau}
\]

Where:
\( \theta \) = Weighting factor (takes values between 0 and 1).
\( \mu_i \) = Average of ST (log-normal distribution i).
\( \sigma_i \) = Standard deviation of ST (log-normal distribution i).
\( \tau \) = Time to option maturity.
\( \alpha_i, \beta_i \) = Log-normal distribution parameters.

The method consists in finding the parameters \( \theta, \alpha_1, \alpha_2, \beta_1 \) and \( \beta_2 \) that best fit the market-quoted option prices with the option valuation formula. But first we need an expression that relates option prices with the function density parameters. We need a formula that can value an option using these five parameters as inputs. For that, we use the expression found by Cox-Ross (1976) where option prices are valued as the discounted expected values of the underlying.
This model yields the call option price $C_t$ at time $t$ as the risk-neutral expected pay-off of the option at expiry $T$, discounted back at the risk-free rate:

$$C(X, \tau) = e^{-r\tau} \int_{-\infty}^{\infty} (S_T - X) f(S_T) dS_T \quad (2)$$

where $S_T$ is the underlying asset price at maturity $T$, $f(S_T)$ is its risk-neutral pdf, $X$ is the option’s strike price and $r$ and $\tau = T - t$ are the risk-free rate and the maturity of the option, respectively. The put price can be recovered either through put-call parity or by replacing the pay-off of the call $S_T - X$ with the pay-off of the put $X - S_T$ in equation and by integrating from zero to the strike price.

When we combine equations (1) and (2) we obtain this (Bahra, 1997):

$$C(X, \tau) = e^{-r\tau} \left[ \theta e^{\alpha_1 + 0.5\beta_2} N(d_1) - XN(d_2) \right] + (1 - \theta) \left[ e^{\alpha_2 + 0.5\beta_2} N(d_3) - XN(d_4) \right] \quad (3)$$

$$P(X, \tau) = e^{-r\tau} \left[ \theta XN(-d_2) - e^{\alpha_1 + 0.5\beta_2} N(-d_1) \right] + (1 - \theta) \left[ XN(-d_4) - e^{\alpha_2 + 0.5\beta_2} N(-d_3) \right] \quad (4)$$

We now have two equations to price call and put functions using the parameters we used before to define the mixture of log-normals. The next step is to find the parameters that minimise the differences between equations 3 and 4 and quoted market prices for call and put options ($C_i$, $P_i$). We minimise differences with respect to the strike price $X$. So, the equation that allows us to find the parameters $\theta$, $\alpha_1$, $\alpha_2$, $\beta_1$ and $\beta_2$, that minimises the differences between observed prices and those obtained from the density function is:

$$\text{Min} \sum_{i=1}^{n} \left[ C(X_i, \tau) - \hat{C}_i \right]^2 + \sum_{i=1}^{n} \left[ P(X_i, \tau) - \hat{P}_i \right]^2 + \left[ \theta e^{\alpha_1 + 0.5\beta_2} + (1 - \theta) e^{\alpha_2 + 0.5\beta_2} - F \right]^2$$

where $F$ is the forward value for $S_T$.

Finally, once we have the parameters $\theta$, $\alpha_1$, $\alpha_2$, $\beta_1$ and $\beta_2$, we can obtain the density function for the underlying using equation 1. This methodology offers better results the broader the range of strikes available.

**The Non-Parametric Method**

The non-parametric technique for estimating fixed expiry-date pdfs—described in Bliss and Panigirtzoglou (2000)—exploits the result derived by Breedon and Litzenberger (1978) that the pdf can be recovered by calculating the second partial derivative of the call price function with respect to the strike price. This result can be derived simply by taking the second partial derivative of the call price function with respect to the strike price $X$, to get:

$$\frac{\partial C^2}{\partial X^2} = e^{-r\tau} f(S_T)$$

So, we just have to adjust the probabilities by $e^{-r\tau}$ to get $f(S_T)$. In practice, we only have a discrete set of strike prices. Thus, to obtain an estimate of the continuous call-pricing function, we need to interpolate across the discrete set of prices. However, the call price function has a large curvature for options near ATM and very little curvature for options OTM. This can make direct interpolation across the call price function difficult.
To avoid this practical problem, we transform the call price function into a particular form of ‘volatility smile’: estimate a smooth smile, convert it back into a call price function and use that to derive the pdf.

To convert a call price function into the relevant volatility smile involves transforming both axes in a non-linear way. We convert option prices into implied volatilities. The implied volatility is the volatility of the underlying asset price implied by the Black-Scholes model and is a non-linear transformation of the option price.

Following Shimko (1993), this interpolation can be done across the volatility smile, using the Black-Scholes formula to transform this back to prices. The reason for doing this, rather than interpolating the call price function directly, is that it is difficult to accurately fit the shape of the latter. And since we are interested in the convexity of that function, any small errors will tend to be magnified into large errors in the final estimated pdf.

Shimko (1993) used a quadratic form to interpolate across the implied volatility smile; others use a cubic smoothing spline. This is a more flexible non-parametric curve that gives us control of the amount of smoothing of the volatility smile and hence the smoothness of the estimated pdf. But following Malz (1997), we should first calculate the Black-Scholes deltas of the options. This is because in practice it is usually easier to interpolate across the volatility smile in ‘delta space’ than in ‘strike price space’. Finally, to generate the implied pdf, we calculate the second partial derivative with respect to the strike price numerically and adjust for the effect of the discount factor.

WHAT’S PRICED IN? A LOOK AT SOME SPANISH STOCKS

Figure 5. Telefonica – Accumulated Probability and Density Function

Note 1: The graph represents the implied pdf for TEF extracted using the mixture of two log-normals (see appendix). The x-axis represents the range of values in the TEF and the y-axis represents the probability attached to each of the values by the options market. The red vertical line represents current prices and the black represents the mode of the distribution (the most likely scenario).

Note 2: The graph represents the accumulated probability for the TEF extracted using the mixture of two log-normals (see appendix). The x-axis represents the range of values in the TEF and the y-axis represents the accumulated probability attached to each of the values by the options market. For example, the options market assigns a 42% probability that TEF will be lower than current prices at year-end, as represented by the red vertical line. Also, €20 is the level that would leave a 50% probability to each side of the distribution at December’s maturity date, as represented by the black vertical lines.

Source: MEFF and Santander Investment Bolsa.
Figure 6. Banco Santander – Accumulated Probability and Density Function

Note 1: The graph represents the implied pdf for the Banco Santander extracted using the mixture of two log-normals (see appendix). The x-axis represents the range of values in the Banco Santander and the y-axis represents the probability attached to each of the values by the options market. The red vertical line represents current prices and the black represents the mode of the distribution (the most likely scenario).

Note 2: The graph represents the accumulated probability for the Banco Santander extracted using the mixture of two log-normals (see appendix). The x-axis represents the range of values in the Banco Santander and the y-axis represents the accumulated probability attached to each of the values by the options market. For example, the options market assigns a 44% probability that Banco Santander will be lower than current prices at year-end, as represented by the red vertical line. Also, €13.6 is the level that would leave a 50% probability to each side of the distribution at December's maturity date, as represented by the black vertical lines.

Source: MEFF and Santander Investment Bolsa.

Figure 7. BBVA – Accumulated Probability and Density Function

Note 1: The graph represents the implied pdf for the BBVA extracted using the mixture of two log-normals (see appendix). The x-axis represents the range of values in the BBVA and the y-axis represents the probability attached to each of the values by the options market. The red vertical line represents current prices and the black represents the mode of the distribution (the most likely scenario).

Note 2: The graph represents the accumulated probability for the BBVA extracted using the mixture of two log-normals (see appendix). The x-axis represents the range of values in the BBVA and the y-axis represents the accumulated probability attached to each of the values by the options market. For example, the options market assigns a 47% probability that BBVA will be lower than current prices at year-end, as represented by the red vertical line. Also, €15.1 is the level that would leave a 50% probability to each side of the distribution at December's maturity date, as represented by the black vertical lines.

Source: MEFF and Santander Investment Bolsa.

Figure 8. Iberdrola – Accumulated Probability and Density Function

Note 1: The graph represents the implied pdf for the Iberdrola extracted using the mixture of two log-normals (see appendix). The x-axis represents the range of values in the Iberdrola and the y-axis represents the probability attached to each of the values by the options market. The red vertical line represents current prices and the black represents the mode of the distribution (the most likely scenario).

Note 2: The graph represents the accumulated probability for the Iberdrola extracted using the mixture of two log-normals (see appendix). The x-axis represents the range of values in the Iberdrola and the y-axis represents the accumulated probability attached to each of the values by the options market. For example, the options market assigns a 33% probability that Iberdrola will be lower than current prices at year-end, as represented by the red vertical line. Also, €10.6 is the level that would leave a 50% probability to each side of the distribution at December's maturity date, as represented by the black vertical lines.

Source: MEFF and Santander Investment Bolsa.
Figure 9. Banco Popular – Accumulated Probability and Density Function

Note 1: The graph represents the implied pdf for the Banco Popular extracted using the mixture of two log-normals (see appendix). The x-axis represents the range of values in the Banco Popular and the y-axis represents the probability attached to each of the values by the options market. The red vertical line represents current prices and the black represents the mode of the distribution (the most likely scenario).

Note 2: The graph represents the accumulated probability for the Banco Popular extracted using the mixture of two log-normals (see appendix). The x-axis represents the range of values in the Banco Popular and the y-axis represents the accumulated probability attached to each of the values by the options market. For example, the options market assigns a 41% probability that Banco Popular will be lower than current prices at year-end, as represented by the red vertical line. Also, €12.6 is the level that would leave a 50% probability to each side of the distribution at December’s maturity date, as represented by the black vertical lines.

Source: MEFF and Santander Investment Bolsa.

Figure 10. Inditex – Accumulated Probability and Density Function

Note 1: The graph represents the implied pdf for the Inditex extracted using the mixture of two log-normals (see appendix). The x-axis represents the range of values in the Inditex and the y-axis represents the probability attached to each of the values by the options market. The red vertical line represents current prices and the black represents the mode of the distribution (the most likely scenario).

Note 2: The graph represents the accumulated probability for the Inditex extracted using the mixture of two log-normals (see appendix). The x-axis represents the range of values in the Inditex and the y-axis represents the accumulated probability attached to each of the values by the options market. For example, the options market assigns a 44% probability that Inditex will be lower than current prices at year-end, as represented by the red vertical line. Also, €41 is the level that would leave a 50% probability to each side of the distribution at December’s maturity date, as represented by the black vertical lines.

Source: MEFF and Santander Investment Bolsa.

Figure 11. Repsol – Accumulated Probability and Density Function

Note 1: The graph represents the implied pdf for the Repsol extracted using the mixture of two log-normals (see appendix). The x-axis represents the range of values in Repsol and the y-axis represents the probability attached to each of the values by the options market. The red vertical line represents current prices and the black represents the mode of the distribution (the most likely scenario).

Note 2: The graph represents the accumulated probability for the Repsol extracted using the mixture of two log-normals (see appendix). The x-axis represents the range of values in the Repsol and the y-axis represents the accumulated probability attached to each of the values by the options market. For example, the options market assigns a 48% probability that Repsol will be lower than current prices at year-end, as represented by the red vertical line. Also, €24 is the level that would leave a 50% probability to each side of the distribution at December’s maturity date, as represented by the black vertical lines.

Source: MEFF and Santander Investment Bolsa.
Figure 12. Gas Natural – Accumulated Probability and Density Function

Note 1: The graph represents the implied pdf for the Gas Natural extracted using the mixture of two log-normals (see appendix). The x-axis represents the range of values in Gas Natural and the y-axis represents the probability attached to each of the values by the options market. The red vertical line represents current prices and the black represents the mode of the distribution (the most likely scenario).

Note 2: The graph represents the accumulated probability for the Gas Natural extracted using the mixture of two log-normals (see appendix). The x-axis represents the range of values in the Gas Natural and the y-axis represents the accumulated probability attached to each of the values by the options market. For example, the options market assigns a 38% probability that Gas Natural will be lower than current prices at year-end, as represented by the red vertical line. Also, €42 is the level that would leave a 50% probability to each side of the distribution at December’s maturity date, as represented by the black vertical lines.

Source: MEFF and Santander Investment Bolsa.

WHO IS WHO IN RNDs

The mean is the expected future value of the underlying or the average value of all possible future outcomes. The median, which has 50% of the distribution on either side of it, is another and alternative measure of the centre of a distribution. The mode is the most likely future outcome.

The standard deviation of an implied RND function is a measure of the uncertainty around the mean and is analogous to the implied volatility measure derived from options prices. An alternative dispersion statistic is the interquartile range (IQR). This gives the distance between the 25% quartile and the 75% quartile, that is, the central 50% of the distribution lies within it.

Skew characterises the distribution of probability either side of the mean. A positively/negatively skewed distribution is one where there is more/less probability attached to outcomes higher than the mean than to outcomes below the mean. Kurtosis is a measure of how peaked a distribution is and/or the likelihood of extreme outcomes; the greater this likelihood, the fatter the tails of the distribution.
BIBLIOGRAPHY


Important Disclosures

This report has been prepared by Santander Investment Bolsa, Sociedad de Valores, S.A. ("Santander Investment Bolsa") and is provided for information purposes only. This document must not be considered as an offer to sell or a solicitation of an offer to buy. Any decision by the recipient to buy should be based on publicly available information on the related security and, where appropriate, should take into account the content of the related prospectus filed with the CNMV (Spanish National Securities Market Commission) and available from the CNMV, the company governing the related market (Sociedad Rectora de la Bolsa) and the company issuing the security. This report is issued in the United States by Santander Investment Securities, Inc. ("SIS"), in Spain by Santander Investment Bolsa and in the United Kingdom by Banco Santander, S.A., London Branch ("Santander London"), which is regulated by the Financial Services Authority in the conduct of its investment business in the UK. SIS, Santander Investment Bolsa and Santander London are members of Grupo Santander. This report is not being issued to private customers.

The information contained herein has been compiled from sources believed to be reliable, but while all reasonable care has been taken to ensure that the information contained herein is not untrue or misleading at the time of publication, we make no representation that it is accurate or complete and it should not be relied upon as such. All opinions and estimates included herein constitute our judgement as at the date of this report and are subject to change without notice. Santander Investment Bolsa may change the recommendation it has on a stock at any given time. It may also cease to cover a stock or initiate coverage of new stocks. There is no specific calendar for any such actions. From time to time, Grupo Santander and/or any of its officers or directors may have a position, or otherwise be interested in, transactions in securities which are directly or indirectly the subject of this report. Santander Investment Bolsa has internal rules of conduct that contain, among other things, procedures to prevent conflicts of interest with respect to recommendations, including: the consideration of its Research Department as a separate area, Chinese Walls, and the possibility of establishing specific restrictions on research activity where appropriate. Santander Investment Bolsa’s research reports contain a certification stating that they reflect the authors’ own opinions.

Grupo Santander may from time to time perform services for or solicit business from any company mentioned in this report. Neither Grupo Santander nor any other person accepts any liability whatsoever for any direct or consequential loss arising from any use of this report or its contents. This report may not be reproduced, distributed or published by any recipient for any purpose. Santander Investment Bolsa is under the supervision of the CNMV.

Any US recipient of this report (other than a registered broker-dealer or a bank acting in a broker-dealer capacity) that would like to effect any transaction in any security discussed herein should contact and place orders in the United States with the company distributing the research, SIS at (212) 692-2550, which, without in any way limiting the foregoing, accepts responsibility (solely for purposes of and within the meaning of Rule 15a-6 under the US Securities Exchange Act of 1934) under this report and its dissemination in the United States. US recipients of this report should be advised that this research has been produced by a non-member affiliate of SIS and, therefore, by rule, not all disclosures required under NASD Rule 2711 apply.


LOCAL OFFICES

Madrid
Tel: 34-91-257-2309
Fax: 34-91-257-1811

Bogotá
Tel: 571-644-8006
Fax: 571-592-0638

Mexico City
Tel: 5255-5629-5040
Fax: 5255-5629-5846

Lisbon
Tel: 351-21-389-3400
Fax: 351-21-387-9133

Buenos Aires
Tel: 54114-341-1052
Fax: 54114-341-1226

Santiago
Tel: 562-336-3300
Fax: 562-697-3869

London
Tel: 44-207-332-6900
Fax: 44-207-332-6909

Caracas
Tel: 582-401-4306
Fax: 582-401-4219

São Paulo
Tel: 5511-5538-8226
Fax: 5511-5538-8407

New York
Tel: 212-692-2550
Fax: 212-407-4540

Lima
Tel: 511-215-8100
Fax: 511-215-8185

Tokyo
Tel: 813-3211-0356
Fax: 813-3211-0362