CBOE Interest Rate Swap Volatility Index (SRVIX^{SM} Index) White Paper

Scope of the Document

This white paper explains the design and properties of the CBOE Interest Rate Swap Volatility Index ("SRVIX[SM] Index" or the "Index") Methodology. The Index Methodology measures the fair market value of future volatility implied by the swaption market for any swaption maturity and tenor of the underlying swap. The Index differs from at-the-money (ATM) implied volatilities as it incorporates additional information contained in the entire skew of out-of-the-money (OTM) swaption prices, serving as the swap rate counterpart to the CBOE Volatility Index® (VIX® Index) for equity volatility. The SRVIX Index is the first interest rate swap volatility index launched by CBOE and is based on 1Y-10Y US Dollar swaptions.

Acknowledgements

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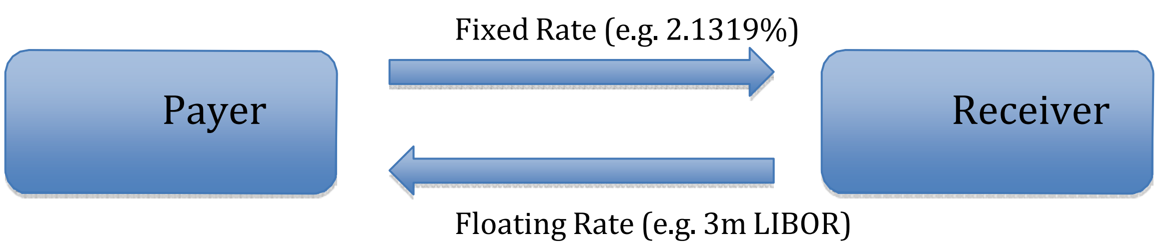
1. Background: Interest Rate Swaps and Swaptions

INTEREST RATE SWAPS constitute the largest segment of the global over-the-counter (OTC) derivatives market with over $400 trillion in outstanding notional at the end of 2011.[1] Swap rates play a central role in the global financial system as they reflect term financing rates of major financial institutions and act as benchmarks for corporate debt funding costs. Moreover, swaps are an important tool for corporate treasurers, asset managers, and public institutions for hedging interest rate exposure. As such, the impact of the level and volatility of swap rates is felt in many corners of the global economy.

The most commonly traded interest rate swaps are known as "vanilla" swaps, in which two counterparties agree to exchange fixed interest payments for floating interest payments on a fixed notional amount over a period of time. In the US Dollar swap market, the floating rate is typically set to the 3-month London Interbank Offered Rate (LIBOR), which is the short-term lending rate between high credit quality banks.



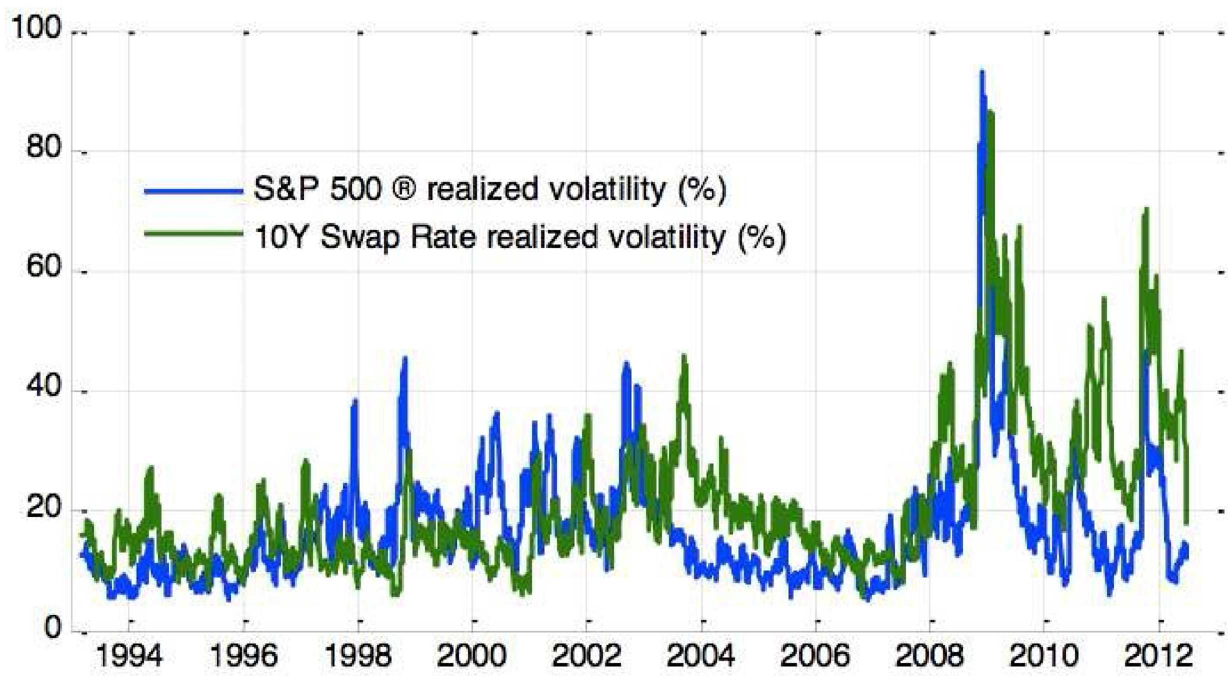
The swap rate is the fixed interest rate in a swap contract, which is determined such that the net present value of the fixed cash flows is equal to the expected net present value of the floating cash flows implied by the forward LIBOR curve. In other words, it is the fair level of compensation paid to the receiver for bearing the risk of paying uncertain short-term rates over the life of the swap.

[1] Source: Bank of International Settlements: "Semiannual OTC derivatives statistics at end-December 2011"

SWAPTIONS are derivative contracts that give the owner the right but not the obligation to enter into an underlying swap contract. A European-style payer (resp. receiver) swaption gives the owner the right at maturity to enter into a swap as the payer (resp. receiver) at a pre-determined fixed rate called the strike. For example, a receiver swaption on a 10-year swap with a $10 million notional and a strike of 2.1319% maturing in 1-year gives the option holder the right 1-year later to enter into a 10-year swap with a notional of $10 million receiving a fixed rate of 2.1319% and paying the 3-month LIBOR. Buyers of swaptions may hedge against or express views on future changes in swap rates while limiting the downside to the premium paid at the time of purchase.

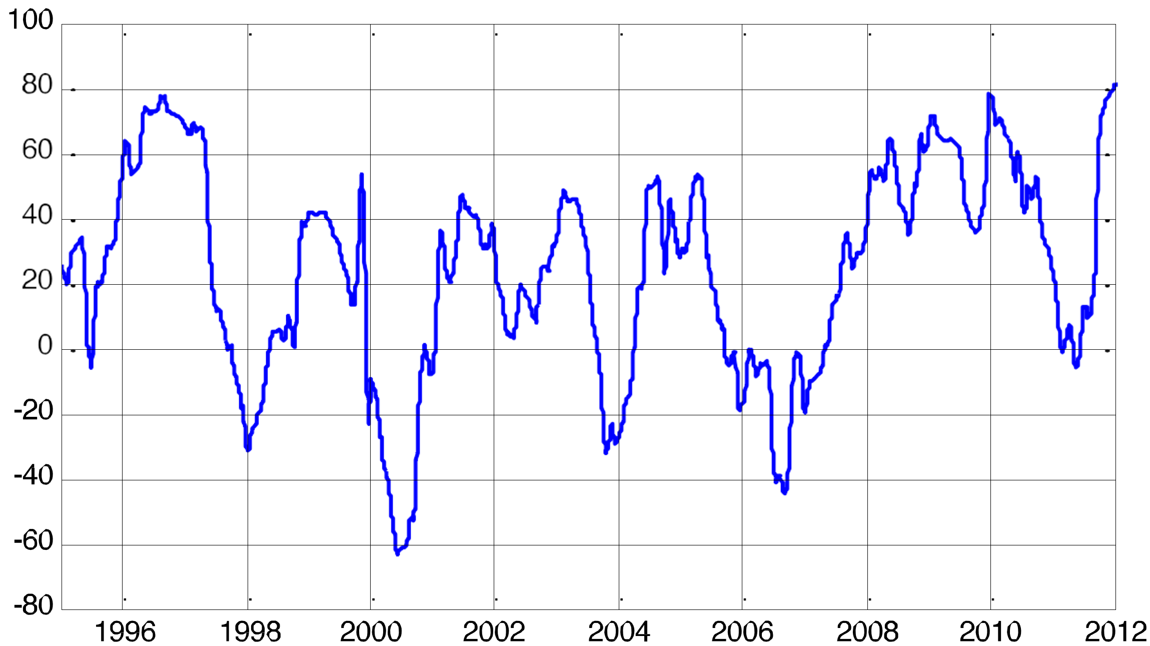
Swaptions are customarily quoted in terms of implied volatilities and converted to premiums using the Black (1976) formula. For any given {maturity, tenor} combination, the range of strikes and their corresponding implied volatilities is called the swaption "skew." The swaption market can be represented as an implied volatility "cube" in which each available {maturity, tenor, strike} combination is mapped to an implied volatility. There is an extra dimension compared to the implied volatility "surface" of equity option markets because the tenor of the underlying swap introduced an additional parameter.

Figure 1: S&P 500® and 10Y Swap Realized Volatility (1M estimation window)



Swap rate volatility has historically behaved differently from volatilities of other major asset classes such as equities. As an example, Figure 1 above shows that while realized volatilities of the S&P 500® index and 10-year swap rates share some common trends, there are periods of significant diversion. Figure 2 moreover shows that the correlation between the two volatility series fluctuates within a wide range over time and averages around 51% over the period shown. These observations suggest that there are drivers unique to swap rate volatility that render it a risk distinct from volatilities of other asset classes.

Figure 2: Correlation between S&P 500 and 10Y Swap Realized Volatility (1Y Est. Window)



2. CBOE Interest Rate Swap Volatility Index (SRVIX Index)

The SRVIX Index Methodology estimates option-implied fair values of expected forward swap rate volatilities for various maturities and underlying swap tenors, and is designed to standardize the way swap market volatility is measured, much as the VIX® Index did for equity markets. For each maturity and underlying swap tenor, e.g. 1-year swaption on a 10-year swap, the Index Methodology aggregates implied volatility information contained in the skew of ATM and OTM swaptions into a single annualized basis point volatility number. The value of each Index is theoretically equivalent to a scaled model-free price of a basis point variance swap contract, which leads to a natural interpretation of the index as the fair market value of forward swap rate volatility.

The SRVIX is the first swap rate volatility index launched by CBOE and is based on ATM and OTM 1Y-10Y USD swaptions.

THE FORMULA of the Index Methodology on date t for maturity of M years and underlying swap tenor of T years is

$$Index_t(M,T) \equiv 100^2 \times \sqrt{\frac{2}{A_t \times M}\left[\sum_{i:K_i<R_t} \mathrm{Receiver}_t(K_i,M,T)\Delta K_i + \sum_{i:K_i \geq R_t} \mathrm{Payer}_t(K_i,M,T)\Delta K_i\right]}$$

where $\mathrm{Receiver}_t(K_i,M,T)$ and $\mathrm{Payer}_t(K_i,M,T)$ are receiver and payer swaption premiums, respectively, with strike rate K_i, maturity M, and underlying swap tenor T; R_t is the ATM strike rate; A_t is the price value of a basis point, also known as the annuity factor, at time t on the fixed leg of a forward T-year swap starting M-years from t; and $\Delta K_i = (K_{i+1} - K_{i-1})/2$ for $i \geq 1$, $\Delta K_0 = (K_1 - K_0)$, and $\Delta K_N = (K_N - K_{N-1})$ where K_0 and K_N are the lowest and highest strikes traded in the market with a total of $N+1$ available strikes. [2]

The SRVIX Index is calculated using the above formula such that $SRVIX_t \equiv Index_t(1,10)$.

THE IMPLEMENTATION of the SRVIX is based on real-time feeds of ATM and skew implied lognormal volatility data from multiple top inter-dealer brokers (IDBs) in the OTC swaptions market who update their volatility cubes throughout each trading day based on the deal flows each observes between the major sell-side dealers. As with most OTC markets, there is no centralized source of unique market-clearing swaption prices; the more trade flow an institution sees, the more its prices are likely to be representative of the overall market. Therefore, aggregating data from multiple IDBs, each of which aggregates information from the major market-makers that further aggregate information from all of their buy-side clients, provides robustness in the price discovery process required for CBOE to calculate and disseminate the SRVIX Index.

[2] A technical document with derivations and other details can be found at

A SAMPLE CALCULATION for the SRVIX Index is given below based on the 1Y-10Y swaption implied volatility data shown in Table 1. The lognormal implied volatility is the volatility parameter σ in the standard swaption pricing formula below based on Black (1976). The corresponding basis point implied volatility—lognormal implied volatility multiplied by the forward swap rate—is also given for reference since the SRVIX is quoted in terms of basis point volatility.

Table 1: Sample 1Y - 10Y Swaption Data

Strike Rate (%)	Lognormal Imp Vol	Basis Point Imp Vol	Rec Prem/A (bps)	Pay Prem/A (bps)
0.1319	41.4	88.26	≈0.00	200.00
0.6319	41.21	87.86	0.02	150.02
1.1319	41.03	87.47	1.67	101.67
1.3819	40.93	87.26	5.15	80.15
1.6319	40.84	87.07	11.61	61.61
1.8819	40.74	86.85	21.38	46.38
1.9319	40.73	86.83	23.73	43.73
1.9819	40.71	86.79	26.20	41.20
2.0319	40.69	86.75	28.79	38.79
2.0819	40.67	86.70	31.51	36.51
2.1319	40.65	86.66	34.34	34.34
2.1819	40.63	86.62	37.28	32.28
2.2319	40.61	86.58	40.33	30.33
2.2819	40.59	86.53	43.48	28.48
2.3319	40.57	86.49	46.74	26.74
2.3819	40.55	86.45	50.09	25.09
2.6319	40.46	86.26	68.16	18.16
2.8819	40.36	86.04	88.04	13.04
3.1319	40.27	85.85	109.32	9.32
3.6319	40.48	86.30	154.89	4.89
4.1319	40.79	86.96	202.63	2.63

Note: 1Yx10Y forward swap rate = 2.1319%

Payer and receiver swaption prices based on the Black (1976) formula are given by:

$$\text{Payer}_t(K,M,T) = A_t \times [R_t \Phi(d_1) - K\Phi(d_2)]$$

and

$$\text{Receiver}_t(K,M,T) = A_t \times [K\Phi(-d_2) - R_t\Phi(-d_1)]$$

for $d_1 = \dfrac{\ln(R_t / K) + (\sigma^2/2)M}{\sigma\sqrt{M}}$, $d_2 = d_1 - \sigma\sqrt{M}$ where σ is the lognormal volatility parameter, Φ is the standard cumulative normal distribution function, and the rest of the variables are as defined in the previous section. Because the A_t in the denominator of the SRVIX formula cancels out the A_t in the Black (1976) formula, we only need to calculate $[R_t\Phi(d_1) - K\Phi(d_2)]$ for payers and $[K\Phi(-d_2) - R_t\Phi(-d_1)]$ for receivers, i.e. premium divided by A_t, which are found in the last two columns of Table 1.

Table 2: Components of SRVIX

Strike Rate (%)	Type	Premium/A (bps)	ΔKi	Prem / A x ΔKi
0.1319	Receiver	≈0.00	0.005	≈0
0.6319	Receiver	0.02	0.005	1.065E-08
1.1319	Receiver	1.67	0.00375	6.24863E-07
1.3819	Receiver	5.15	0.0025	1.28793E-06
1.6319	Receiver	11.61	0.0025	2.90358E-06
1.8819	Receiver	21.38	0.0015	3.20658E-06
1.9319	Receiver	23.73	0.0005	1.18649E-06
1.9819	Receiver	26.20	0.0005	1.31003E-06
2.0319	Receiver	28.79	0.0005	1.43971E-06
2.0819	Receiver	31.51	0.0005	1.57537E-06
2.1319	ATM	34.34	0.0005	1.71683E-06
2.1819	Payer	32.28	0.0005	1.6139E-06
2.2319	Payer	30.33	0.0005	1.5164E-06
2.2819	Payer	28.48	0.0005	1.42413E-06
2.3319	Payer	26.74	0.0005	1.33687E-06
2.3819	Payer	25.09	0.0015	3.76332E-06
2.6319	Payer	18.16	0.0025	4.53953E-06
2.8819	Payer	13.04	0.0025	3.2598E-06
3.1319	Payer	9.32	0.00375	3.49406E-06
3.6319	Payer	4.89	0.005	2.44285E-06
4.1319	Payer	2.63	0.005	1.31425E-06
			SUM	3.99671E-05

Table 2 lists the relevant ATM and OTM swaption premiums divided by A_t and the corresponding values of ΔK_i. Multiplying the two and summing across all rows yields 3.9967E-5, which leads to the index value

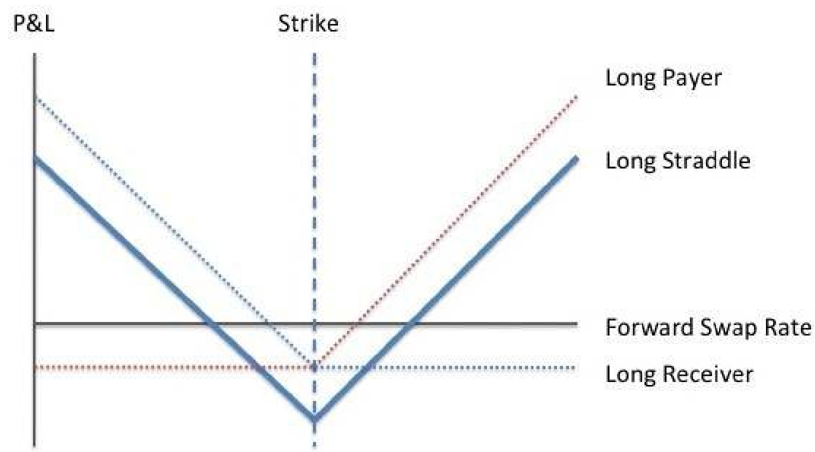
$$SRVIX_t = Index(12/12,10) = 100^2 \times \sqrt{\frac{2}{(12/12)} \times 3.99671\text{E-}05} = 89.40$$

which is higher than the ATM implied basis point volatility of 86.66, although this need not always be the case depending on the shape of the skew.

3. Volatility Trading

VOLATILITY TRADING in the swap market is commonly done through delta-hedged ATM swaption straddles. In a straddle, a trader buys (resp. sells) a payer and a receiver with the same maturity, underlying tenor, and strike to go long (resp. short) volatility of the forward swap rate.

Figure 3: Payoff Diagram of an ATM Straddle



Once initiated, straddles must be dynamically hedged for the strategy to remain insensitive to movements of the underlying forward swap rate, i.e. delta-neutral.

FUTURES AND OPTIONS on CBOE Interest Rate Swap Volatility Indexes[3] would augment existing volatility trading practices by allowing market participants to capture pure volatility exposure using standardized contracts with payoffs directly linked to movement in the indexes. Potential uses include:

- Expressing directional views on implied volatilities of specific maturity/tenor points
- Expressing relative value views on implied volatilities between multiple maturity/tenor points
- Expressing relative value views between ATM implied volatilities and skew implied volatilities
- Hedging fixed income portfolios against the impact of large volatility moves on fixed income asset prices
- Hedging against decrease in market liquidity often accompanying volatility spikes
- Hedging volatility exposure of structured trades and products

[3] As of the date of this paper, these products are not available for trading.