Management of Asian and Cliquet Option Exposures for Insurance Companies: SPVA applications (I)

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5th September, 2014, Wicklow, Ireland

Genossenschaftliche FinanzGruppe
Volksbanken Raiffeisenbanken

Agenda

1. Introduction
2. Review of Asian option and Cliquet option
3. SPVA in current markets
4. Product innovation
5. Pricing issues on new product
6. Hedging issues on new product
7. Conclusions

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1. Introduction

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2. Review of Asian option and Cliquet option

Asian Options

In a nutshell

- an exotic option.
- the payoff is path dependent.
- the payoff depends on the average price of the underlying over a predetermined period of time.

The most general definition of an Asian option, which is also known as the "**Average Option**", there are a few different possibilities within the option.

2. Review of Asian option and Cliquet option

Average

- The average can be either arithmetic or geometric.

- Arithmetic Asian option:
  - No closed form solution exists.
  - Numerical methods, e.g., Monte Carlo simulation, have to be used.

- Geometric Asian option:
  - Exists a closed form solution due to the good properties of a geometric mean embedded in the stock stochastic process.

- The option could vary between being an average price option or an average strike option.

2. Review of Asian option and Cliquet option

Average (continued)

- The payoff for the average price call option is $\max\{0, A(T) - K\}$.
- The payoff for the average strike call option is $\max\{0, S_T - A(T)\}$.

Where:
T = Time of Maturity
$A(T)$ = Average price of the underlying at time T
K = Fixed Strike
S_t = Price of underlying at time t

2. Review of Asian option and Cliquet option

Period of Time

- over the whole period of time,
- over the last several days, or
- over a select amount of dates agreed upon at the time of inception.

Strike

- fixed, which is decided before the contract, or
- floating, decided at maturity.
  - floating strikes tend to be path dependent, which rely on the underlying performance during the option life.

Style

- Most Asian options are European style, they can be exercised at maturity.
- Some Asian options are American style, they can be exercised anytime and the payoff is an up-to-date average.

2. Review of Asian option and Cliquet option

How to price it?

- We will look at the formula for an European style Asian option with a geometric average and a fixed strike.
- This is found using PDE and stochastic calculus (See Appendix).

There are many other ways to conduct the pricing exercises:

- Monte-Carlo simulation.
- Binomial tree.
- Convolution method.
- Direct integration.
- Partial differential equation (PDE).
- Fourier transform (FFT).
- Approximate analytic method.

2. Review of Asian option and Cliquet option

When to use it?

- Foreign exchange markets:
  - it is less risky to deal with an average over a period of time then to rely on the rate at maturity.

- Avoid price manipulation:
  - if the price can be easily manipulated at a single point in time then it is safer and less risky to use the average.

- Thin markets:
  - in markets with low trading volumes or illiquid assets, it is better to use the average than the spot price as any abrupt change in supply and demand will have a larger than proportional impact on prices.

2. Review of Asian option and Cliquet option

Applications to Life Insurance and Annuities

- Using foreign exchange as an underlying asset, or
- Using commodity products as underlying asset (MetLife SLS, AXA SCS), or
- Using crude oil futures and options on the NYMEX (AXA SCS).

2. Review of Asian option and Cliquet option

Cliquet Option

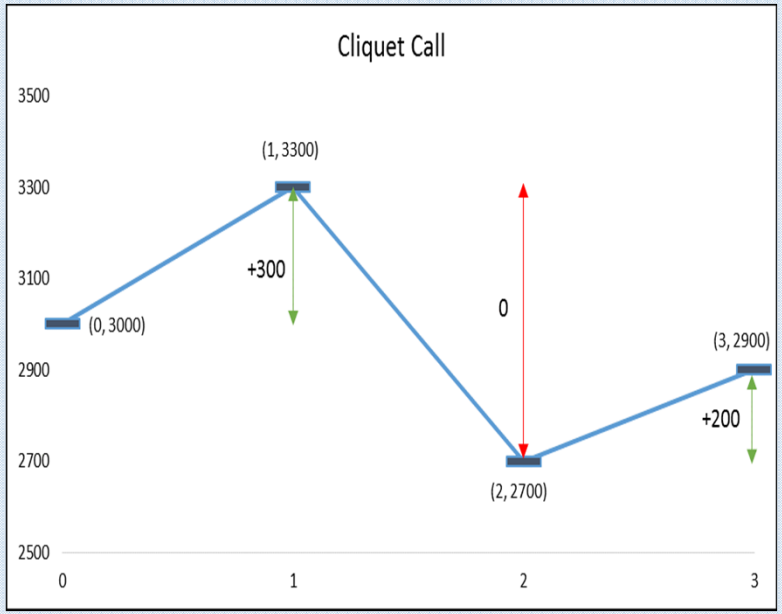
Also known as a **Rachet option** or a series of Forward Start options.

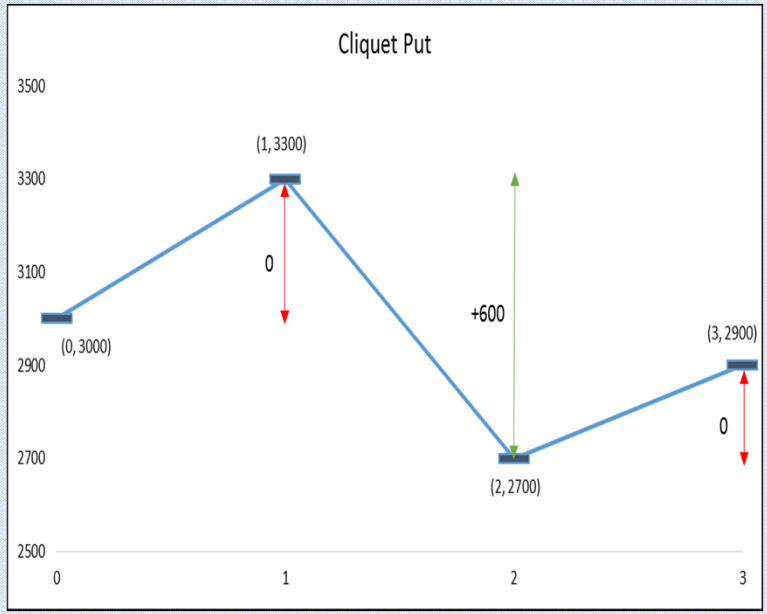
In a nutshell

- An option whose strike resets periodically.
- The reset is done at predetermined dates. At this date, the option settles and the strike is reset to the current spot level.
- It is similar to a series of at-the-money options purchased for a known price at inception.
- The payout from each of these at-the-money options is either collected at the end of that period or at final maturity.

2. Review of Asian option and Cliquet option

Example of Cliquet Call and Cliquet Put





Cliquet Call vs European Call:
500 vs 0

Cliquet Put vs European Put:
600 vs 100

2. Review of Asian option and Cliquet option

How to Price It?

- This version of the cliquet option is very easy to price because it can be seen as a series of pre-purchased at-the-money options. See appendix of Rubinstein (1991) approach.
- The above closed form solution is criticised though for its lack of flexibility, if any of the assumptions change then a whole new solution needs to be found.
- How the volatility curve is found is quite important due to the impact it has on the pricing.
- If Company is not using the "right" volatility curve, it is easy to misprice the cliquet option.

2. Review of Asian option and Cliquet option

When to Use It?

An alternative strategy to buying a cliquet option would be to buy an at-the-money option at the start of each equivalent reset date.

Though if volatility is rising this increases the price of the at-the-money options, yet the price of the cliquet is set at the start of the period.

Thus it is best to use cliquet options in a market where volatility is rising.

The cost of volatility is fixed then for the period of the option.

2. Review of Asian option and Cliquet option

Application to Life Insurance and Annuities

- This type of option could be used for any products with a withdrawal benefit, as policyholder can receive its payout throughout the lifetime of the option, not just at its expiration.

- Though not for companies that have dynamic hedging in place as the option is seen more to be for medium term passive investors.

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3. SPVA in current markets

- New variable annuity product which is based on structured product-like investments instead of the mutual fund-like investments.
- Embedding a structured product into a variable annuity.
- Describe structured product based variable annuity (SPVA):
  1. crediting formulas;
  2. differ from traditional VAs;
  3. value the embedded derivative position;
  4. the fair cap levels.
- *The complexity of spVAs can be used to hide fees and reduce the comparability of variable annuities to other investments in the market.*
- Similar products are also named as follow: Structured-Note Annuity(SNAs).

3. SPVA in current markets

- Traditional variable annuities provide a combination of stock and bond portfolios called “subaccounts".
- The value of a variable annuity changes with changes in the net asset value of the subaccounts an investor has selected.
- Structured products are derivative debt securities whose payouts are based on the price changes of a reference asset (index, equity, interest rate, volatility) subject to buffers against losses and caps on gains.
- The value of spVAs is sensitive to price fluctuations of their underlying reference asset and to contract parameters such as the buffer, cap and term.
- The spVAs are more complex than traditional variable annuities, and that cap levels for most underlying assets and time periods must be quite high to fairly compensate investors for the embedded derivatives position they are sold.

3. SPVA in current markets

There are three structured product based variable annuities available to US investors:

1. AXA Equitable's Structured Capital Strategies Variable Annuity (`SCS', first issued in October 2010),
2. MetLife's Shield Level Selector Single Premium Deferred Annuity (`SLS', first issued in May 2013), and
3. Allianz Life's Index Advantage Variable Annuity (`IA', first prospectus dated August 2013).

3. SPVA in current markets

Exposure Types

- Under an spVA investors can choose to allocate premiums to "**segments**" rather than traditional variable annuity subaccounts.
- Each spVA segments has a defined time period (`**term**') that determines when the **underlying asset** is initially observed and when the underlying asset is observed to determine index credits.
- At the end of an spVA segment, the issuer credits the variable annuity account value with a return based on an underlying asset.
- For all spVA segments, returns are calculated *only at maturity*, not periodically over the term of the segment.
- All payout formulas use price returns, not total returns, effectively excluding any potential income from dividends.

3. SPVA in current markets

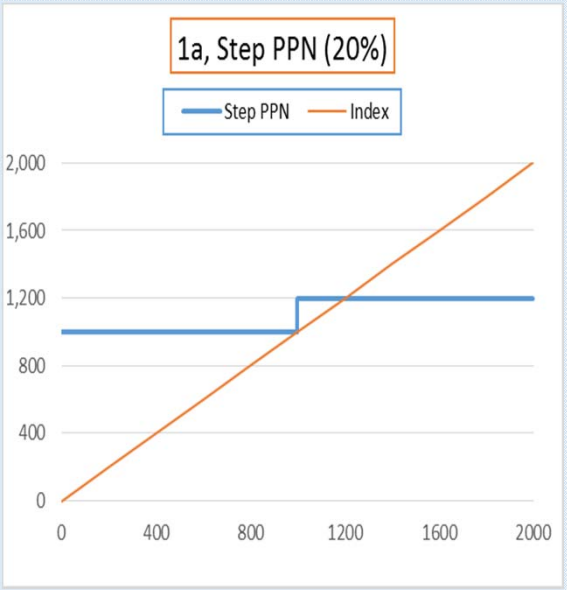
Table 1: Currently Available spVA Underlying Assets

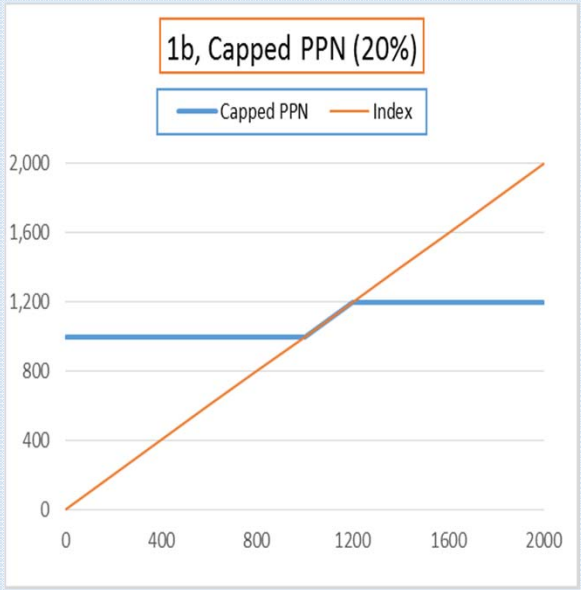
(Note below: Gold, Oil Futures, Real Estate and Commodity Index as underlying)

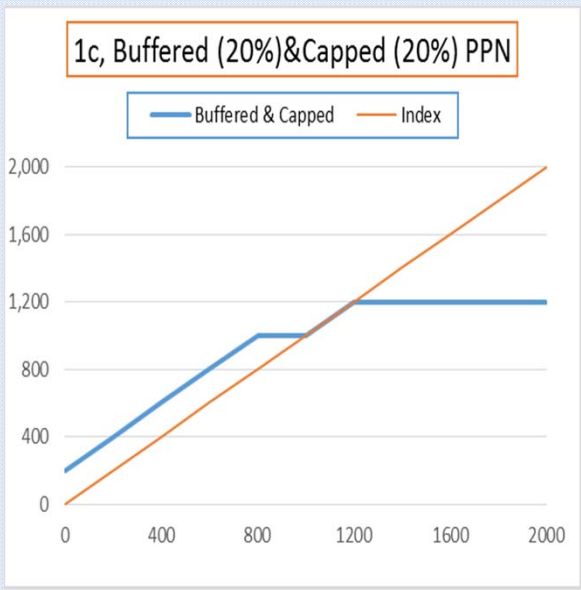
Asset Name	AXA SCS	MetLife SLS	Allianz IA
S&P 500	✓	✓	✓
NASDAQ 100	✓	✓	✓
Russell 2000	✓	✓	✓
MSCI EAFE Index	✓	✓	×
MSCI Emerging Markets Index	✓	×	×
London Gold Market Fix Price (USD)	✓	×	×
NYMEX West Texas Int. Crude Oil Front-Month Futures	✓	×	×
iShares Dow Jones US Real Estate Index Fund	✓	×	×
Dow Jones-UBS Commodity Index	×	✓	×

3. SPVA in current markets

- spVA crediting formulas have been of two types:
  1. a principal protected note like payout (1a, Step PPN; 1b, Capped PPN), or
  2. a buffered and capped payout (1c).







3. SPVA in current markets

PPN-like Structures

- **Step PPN segment** as shown in Figure 1(a), pay a fixed amount (the `step rate') if returns on the underlying asset are positive and otherwise return the amount invested.
  - It can be valued as a zero coupon bond plus a binary option on the underlying.
  - MetLife's Step Rate option and Allianz's Index Protection Strategy, both are available for the S&P 500 index.
- **Capped PPN segment** as shown in Figure 1(b), includes a segment with a 100% buffer, if the reference asset has a negative return at maturity, it returns principal paid, and for any positive returns at maturity, it pays the return up to a cap.
  - MetLife's SLS product; available on the S&P 500 index.

3. SPVA in current markets

Buffered and Capped Structures

- One spVA segment includes a buffer on losses below a certain threshold (the buffer level) selected by the investor, and a cap on returns above a separate level (the cap) selected by the issuer.
- In Figure 1(c), this structure exposes the investor to losses beyond the buffer level, such that an investor's maximum possible loss of value on the segment equals 100% minus the buffer level (@10% buffer, the maximum loss is 90%).
- Some issuers do not disclose the cap level until the segment start date, making it difficult to evaluate a product ex ante.
- Some issuers do not publish historical cap rates, which vary over time and with different buffer levels and underlying assets.

3. SPVA in current markets

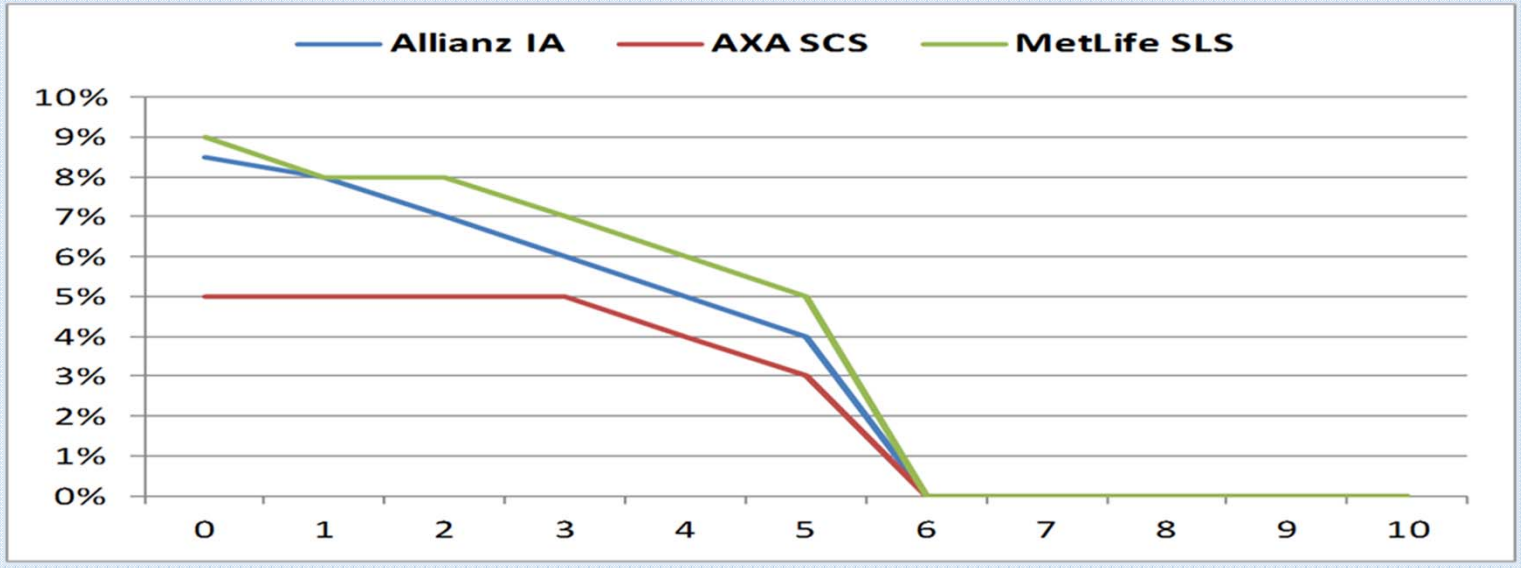
Withdrawals Before Maturity

- Issuers discourage spVA investors from withdrawing their account value by applying a surrender charge or withdrawal fee.
- Surrender charges can be quite large in early years, up to 9% for spVAs, then diminish over time.
- These charges reduce the amount of an investor's account value that s/he can liquidate at a particular point in time.
- Because spVAs are linked to complex derivative positions rather than simple mutual fund-like subaccounts, calculating an account value on any day prior to maturity is complicated and subject to certain assumptions.

3. SPVA in current markets

Surrender Charge Schedule

- VA issuers deduct a surrender charge when customers withdraw funds “to reimburse us for contract sales expenses, including commissions and other distribution, promotion, and acquisition expenses".
- Figure 2 illustrates the surrender charge schedule for each of the three spVAs as a function of years since contract purchase.



3. SPVA in current markets

- Each contract has a provision that allows policyholders to withdraw a portion of their account value without incurring a withdrawal fee.
  - AXA: penalty-free withdrawals up to 10% per year **based on account value.**
  - Allianz: penalty-free withdrawal up to 10% per year **based on premiums paid.**
  - MetLife: penalty-free withdrawal up to 10% per year **based on the most recent policy anniversary's interim value.**
- Withdrawal amounts beyond 10% during the accumulation period of each annuity are charged as depicted in Figure 2.

3. SPVA in current markets

Interim Account Value Calculation

- The first approach:

  To pro-rates the cap and/or buffer level based on the fraction of time that has elapsed in the segment's term since the segment start date.

  - If three months had passed on a 1-year segment, then the cap and/or buffer would be multiplied by ***25%*** and then applied to the returns on the index as of that date.

- The second approach:

  To value the underlying options position using option pricing models.

  - The issuer breaks the segment down into its component options, values each option separately, then adds those values (and the value of the embedded zero-coupon bond) to arrive at a total value of the position.
  - The spVA segments are based-on European-style options, they can be valued using the Black-Scholes options pricing model and current market data.

3. SPVA in current markets

Interim Account Value Calculation

- Allianz's IA annuity follows the second approach, though they also credit interest representing “the value of amortizing the cost of" the option positions over the current policy year.
- AXA compares the calculated value from the prorated cap to the value obtained with the second approach and takes the lesser of the two values. *AXA does not pro-rate the buffer level, only the cap rate.*
- MetLife's SLS uses the first approach, pro-rating both the cap rate and the buffer level.

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8. References

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9. Disclamers and Acknowledgement

- Materials presented above are personal opinions only, I own all errors.
- Thanks to Sinéad McCann for research assistance.
- Pin Chung, CFO and CIO, R+V International Business Services Limited, pin.chung@ruv.ie

Appendix: Asian option with geometric average & a fixed strike

When the boundary condition $v\left(T, S(T), Y(T)\right) = max\left\{0, \frac{Y(T)}{T} - K\right\}$ is applied it is found that there is only one solution for the PDE

$$v_t(t,x,y) + r * v_x(t,x,y) + \frac{1}{2}\sigma^2 * x * v_{xx}(t,x,y) + x * v_y(t,x,y) = r * v(t,x,y)$$

Where

$x = S(t)$

= Price of asset at time t

$y = Y(t)$

$= \int_0^t S(u)du$

T = Time of Expiration $\Rightarrow 0 \leq t \leq T$

σ = Volatility of asset

r = Rate of interest

} Constant

Appendix: Asian option (continued)

The price of the underlying asset is given by

$$S(t) = S(0)exp\left\{\int_0^t \sigma(u)d\widetilde{W}(u) + \int_0^t \left(R(u) - \frac{1}{2}\sigma^2(u)\right)du\right\}$$

Where

$$\widetilde{W}(t) = W(t) + \int_0^t \theta(u)du$$

$$\theta(t) = \frac{\alpha(t)-R(t)}{\sigma(t)}$$ ← Adapted Stochastic Process

$\alpha(t)$ = Instantaneous mean rate of return
$R(t)$ = Rate of interest
$\sigma(t)$ = Volatility of asset
$W(t)$ = Brownian Motion

} Time dependent

This is proved in detail in the paper "Pricing The Asian Call Option" [3]. It is also found that the perfect hedging portfolio is given by

$$\Delta t = v_x(t, S(t), Y(t))$$

Δt = Number of shares of the asset held at time t.

Appendix: Cliquet option (Rubinstein 1991)

Cliquet Option: How to Price It?

The first version of the cliquet option is easy to price because it can be seen as a series of pre-purchased at-the-money options. For the Rubinstein (1991) approach its call is given by the sum of forward start calls.

$$\sum_{i=1}^{n} S e^{-D t_i} \left[e^{-D(T_i - t_i)} N(d_1) - e^{-r(T_i - t_i)} N(d_2) \right]$$

$$d_1 = \frac{\left(r - D + \frac{1}{2}\sigma^2\right) * (T_i - t_i)}{\sigma \sqrt{T_i - t_i}}, \qquad d_2 = d_1 - \sigma \sqrt{T_i - t_i}$$

S = Spot price of asset
T_i = The i^{th} maturity time
t_i = The i^{th} time to forward
D = Continuous dividend rate
r = Risk-free interest rate
σ = Volatility of the asset
$N()$ = The cumulative Normal distribution function