MANAGEMENT OF ASIAN AND CLIQUET OPTION EXPOSURES FOR INSURANCE COMPANIES: SPVA APPLICATIONS (II)
The insurance industry witnessed a significant retrenchment in VA activity:
  - Hartford and Sun Life completely withdrew last year
  - ING and John Hancock greatly reduced the size of their VA books

Hedging still challenging, remember major losses experienced during and after 2008

SPVA, provided they are appropriately designed, is a potential promising response to the problems the VA industry went through in the recent years:
  - Complexity of the VA programs created hedging issues (especially long dated vega footprint)
  - Competitiveness of pricing narrowing margins and fees not consistent with the risks

Interesting market context
  - Low rates (QE) and volatility (VIX at the lowest), hence low yield and risk environment
  - Strong regulatory changes that are reshaping financial markets
PRODUCT INNOVATION
SPVA DESIGN

✓ SPVA offers:
  o Ability to create diverse payout profiles to meet client’s needs, customize risk/reward appetites through adapted structuring mechanisms
  o This is done through derivatives

✓ Popular derivatives or derivatives strategies
  o Averaging/Asian based payout
  o Digital/Barriers based payout
  o Cliquet/Ratchet based payout
  o Risk Controlled Strategies based payout:
    ▪ CPPI
    ▪ Volatility Target (the strategy can be based on various indicator: underlying asset volatility, volatility indices …)
  o Other more complex types

✓ The risks embedded in the above list should managed and hedged
PRICING & HEDGING
TWO SIDES OF THE SAME COIN

✓ The valuation of any risky asset/claim is based on replication arguments
✓ Pricing = Margins + Hedging Cost

Insurance Firms Issuing SPVAs

- **Objective**
  - Create products that will generate a locked-in PnL with no sensitivity to market moves

- Risk neutral valuation

- Hedging (importance of market parameters)

- Hedgeable risks

- Risk neutral based valuation models

Hedging Risk Dichotomy

- **Complete hedging**
  - Risk offset by trading in a set of hedging instruments that provide a faithful representation of the OTC / exotic derivative

- **Incomplete hedging**
  - Risk that you can only partially hedge. The hedging is suboptimal in the sense that the firm will bears some part of the risks
PRICING & HEDGING CHALLENGES & ISSUES

Challenges (related to insurance firms’ hedging programs) to keep in mind in product design phase:

- High cost and limited availability of long dated Vega static hedges
- Basis risk of long dated hedges
- Volatility of dynamic hedging costs
- Volatility of reserve and capital costs (solvency requirements) due to large long dated Vega exposure of guarantees
- Limited Vega offset available from short dated hedging instruments (more cost effective and easier to roll)
- Pricing technique/technology and expertise when it comes to risk that are difficult to hedge, as well as other complex risks embedded in the derivatives used to create products (one-way risk, volatility of volatility …)
PRICING & HEDGING CHALLENGES & ISSUES: ILLUSTRATIONS

- Illustration 1: Averaging based payout
  - Asian option embedded
  - S&P500
  - Mostly Vega risk
  - Pricing models and challenges
  - Hedging Issues

- Illustration 2: Cliquet/Ratchet based payout
  - Cliquet option embedded
  - S&P500
  - Vega and Volga risk
  - Pricing models and challenges
  - Hedging Issues

- Illustration 3: Control Risk Strategy based payout
  - Volatility Target Strategy
  - SX5E
  - Drastically reduced Vega risk
  - Pricing models and challenges
  - Hedging Issues
Pricing & Hedging Challenges & Issues: Illustration 1

Asian option embedded

S&P500

Mostly Vega risk

Pricing models and challenges

Hedging Issues

- No closed form formulas: approximation or stochastic model solutions
- Curran’s approximation: works well except when there are significant volatility shifts specifically in the period before the averaging. In that case underestimation can be very significant for OTM and ITM options
- Local Volatility (LV):

\[ \frac{dS_{LV}(t)}{S_{LV}(t)} = \mu(t)dt + \sigma_{LV}(t, S_{LV}(t)) \ dW(t) \]

- Vega hedging can be done in an efficient way (we are here in the « complete hedging » case) if one uses LV model
- The only issue is that you are exposed to long dated vega
- Being long vega would not be a problem if one had a buyer, but only top tier Sell-Side institutions are big enough to be able to recycle long vega (through variance swap market)
Pricing & Hedging Challenges & Issues: Illustration 2

Pricing models and challenges

- Local volatility (LV): not a good candidate at all because the dynamic is wrong for forward volatility dependent products (see slides on LV dynamics)
- Stochastic Volatility (SV):
  - Heston type: does a better job in terms of the dynamic but has its own problems (see slides on SV models)
  - Two-factor stochastic volatility framework (Bergomi, SG): a new approach that improves on Heston drawbacks
- Stochastic Local Volatility (SLV)

Hedging Issues

- Vega hedging is far more complicated as there is additional risk due to Volga (Vega of vega or also called Vol Gamma)
- Volga: Volga risk explained for a capped cliquet
- One is still exposed to long dated Vega but now with additional second order risk (Volga)
- All of the above is made worse because of diminished supply of long-dated vega
PRICING & HEDGING
ILLUSTRATION 2: LV DYNAMICS

One can see below that volatility itself is very volatile. This feature is essential when dealing with forward volatility dependent products such as cliquets, unfortunately LV dynamics cannot capture it.
One can see below why Local Volatility dynamic is wrong for cliquet/ratchet related products. It systematically underestimates the true volatility risk.
Heston model:

\[
\frac{dS(t)}{S(t)} = \mu(t)dt + \sqrt{v(t)} \, dW(t)
\]

\[
dv(t) = k\left(\theta^2 - v(t)\right)dt + \sum \sqrt{v(t)} d\left(\rho W(t) + \sqrt{1-\rho^2} B(t)\right)
\]

Problems with this model:

✓ Does not fit vanillas well on the short end of the surface
✓ Variance is not guaranteed to be positive (violation of the Feller condition) making propagation unstable
✓ Estimated volatility of volatility not consistent with historical volatility of volatility
PRICING & HEDGING
ILLUSTRATION 2: SLV DYNAMICS

Stochastic Local Volatility model:

\[
\frac{dS(t)}{S(t)} = \mu(t) dt + f(\theta(t), S(t)) \sqrt{v(t)} \, dW(t) \\
\frac{dv(t)}{v(t)} = -kv(t) dt + \Sigma d \left\{ \rho W(t) + \sqrt{1-\rho^2} B(t) \right\}
\]

Advantages with this model:
- Fits the Vanilla surface very well
- Realistic dynamics of Implied Volatility
- Takes into account Vega hedging cost
- Can be fitted to variance swaps and implied volatility

One drawback:
- Heavy numerical calibration
**PRICING & HEDGING CHALLENGES & ISSUES: ILLUSTRATION 3**

**Volatility Target Strategy**
- Drastically reduced Vega risk

**Basic idea of the product**
- Follows a multi-asset strategy and allocates between a risky equity asset and a risk-free asset (where investment amount can be shifted to whenever equity markets are in stressed conditions)
- The trigger to shift is defined by a volatility indicator (volatility of the asset itself, VSTOXX index …)

**Pricing models and challenges**
- Simple Black-Scholes will do a good job due to the fixed level of volatility (close to constant volatility by virtue of the target)
- Volatility has to be estimated in a conservative way (can be done by estimating the standard deviation of the volatility around the target and shifting accordingly)
- Jump model can be added to assess jump risks

**Hedging**
- Target volatility indices highly suitable as underlying for option-based derivative products since the issuing entity faces nearly no Vega risk due to fixed level of volatility
- In period of huge distress, jump risk might be dealt with proxy hedging
Below an illustration of a volatility target strategy @8.5%, visually one understands why Vega risk is insignificant.
Another variation here, where target is \( T = 20\% \), the corresponding target portfolio weight for SPX is \( W = T / \text{VIXX} \) and maximum leverage is \( @ 150\% \).

Performance of the investable target strategy (blue) versus SPX (yellow)
Proxy hedging for jump risk can be done volatility indices (VIX, VSTOXX).
Below, tradable option market is developing rapidly on VIX.