DOUBLE THE FUN WITH CBOE's VVIX^{SM} Index

Introduction

CBOE is expanding its suite of volatility benchmarks with a new index called the VVIX Index, the VVIX for short. The VVIX is a volatility of volatility measure in that it represents the expected volatility of the 30-day forward price of the CBOE Volatility Index (the VIX®). It is this expected volatility that drives the price of VIX® nearby options.

The VVIX is calculated from the price of a portfolio of liquid at- and out-of-the-money VIX® options. The calculation method is the same as for the VIX® and summarized in Appendix 1. CBOE also calculates a term structure of the VVIX associated with different VIX option expirations. The VVIX is disseminated in real time and its term structure is posted on CBOE's web site at

http://www.cboe.com/micro/VVIX/documents/vixvixtermstructure.xls

The VVIX is designed to guide and inform the growing number of investors in VIX-based products. The VVIX and its term structure convey:

- The expected volatility that determines VIX® option prices.
- The expected volatility of the VIX® itself to a nearby horizon.
- The mean and standard distribution of settlement values of VIX® futures and options.

Different points on the VVIX term structure price portfolios of VIX® options (VVIX portfolio) to different expirations. A position in a VVIX portfolio replicates the volatility of VIX® forward prices. VVIX portfolio prices have usually been at a premium relative to future realized volatility. The discount is a volatility risk premium. For nearby expirations, these prices have also tended to surge at the same time as VIX®. These features suggest several trading opportunities:

- Buying a VVIX portfolio returns the difference between realized and expected volatility less the volatility risk premium. Conversely selling a VVIX portfolio returns the difference between expected and realized volatility plus the volatility risk.
- To the extent that volatility expectations are unbiased, consistently selling a VVIX portfolio captures the volatility risk premium.
- Buying a short-dated VVIX portfolio can cushion losses from extreme increases in VIX® futures prices.

1. Historical Behavior of VVIX June 2006 – February 2012

Chart 1a. The VIX® and VVIX, Time Series

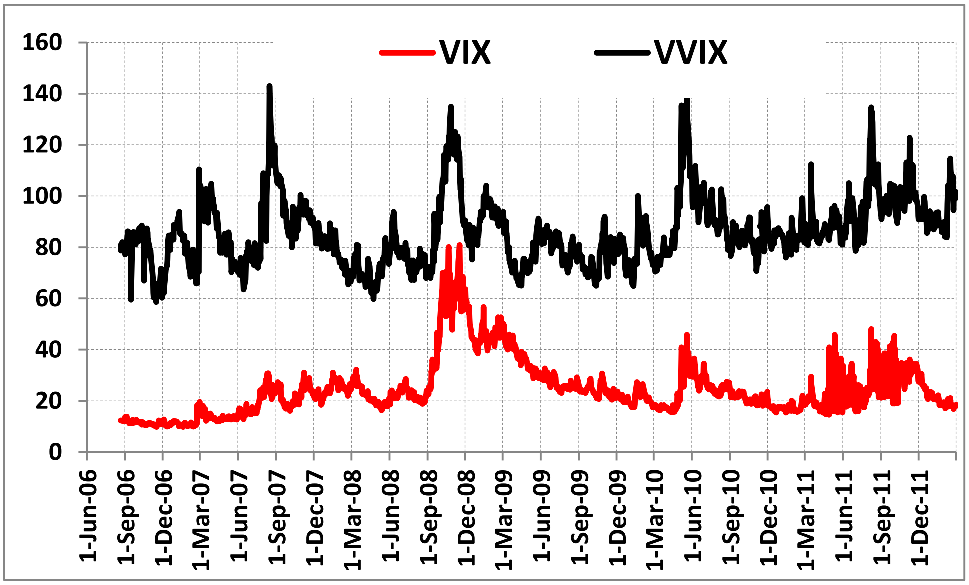


Chart 1b. The VVIX versus the VIX® Sorted in Ascending Order

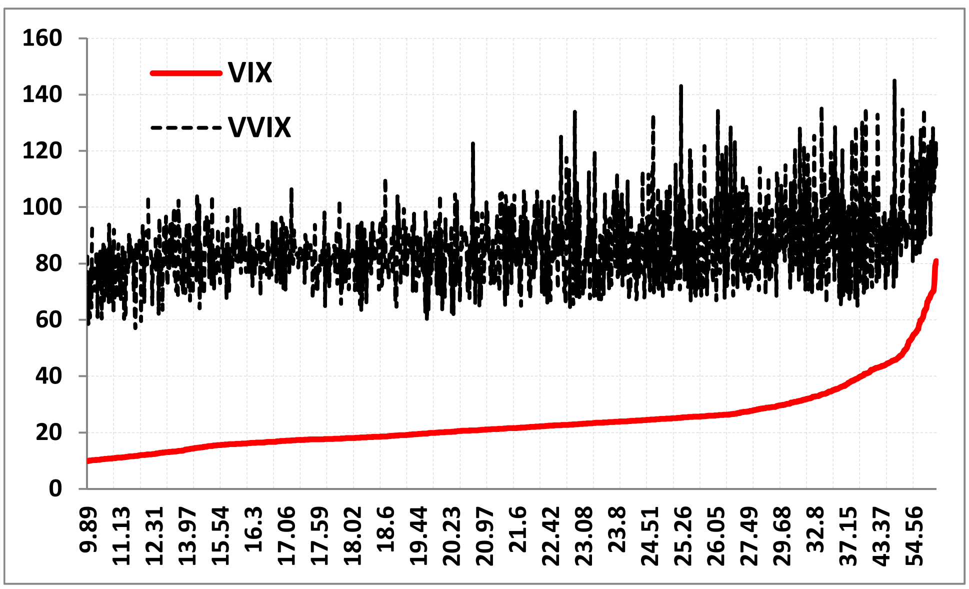


Chart 1a. is a graph of the time series of the VVIX and VIX between June 2006 and February 2012 ; Chart 1b.shows the variation of the VVIX when the VIX® is

sorted in ascending order. Several features are apparent from these charts and from an analysis of index returns:

- **The range of values of the VVIX is at a significantly higher level than that of the VIX®.**
  The VVIX ranges between 60 and 145 around an average of 86. The VIX ranges between 10 and 81 around an average of 24. Also note that the range of variation of the VVIX tends to widen at higher values of the VIX.

- **Except at high values of VIX, there is little correlation between variations of the VIX® and VVIX.**
  The VVIX and VIX® both reached local peaks in October 2008, during the credit crisis of 2008 and in May 2010, the "flash crash" month. In general however, the relationship between their variations is weaker than the relationship between the VIX® and the S&P 500.

- **Since the flash crash of May 2010, the VVIX has rarely dropped below 80.**
  This suggests that a new volatility regime came about after the flash crash. Market participants appear to have become more tentative about the future value of the VIX®.

The following additional features of VVIX characterize its term structure and its tendency to mean revert.

- **The VVIX tends to revert to its historical mean.**
  Since the VVIX is a rate variable, it is mean reverting. Table 1 shows the historical value of the VVIX 30, 60 and 90 days later, starting from different initial intervals. For example, when the initial value of the VVIX is between 30 and 50, its average value 30 days later is 78. The mean reversion is centered about the historical mean of VVIX, approximately 85 and the fixed point is the interval from 80 to 90 volatility points. Starting from this interval, the VVIX ends in the same interval. Starting from an initial value below its historical mean, the VVIX tends to end in a higher range in subsequent months. Conversely, from an initial value above its historical mean, the VVIX tends to revert to a lower range.

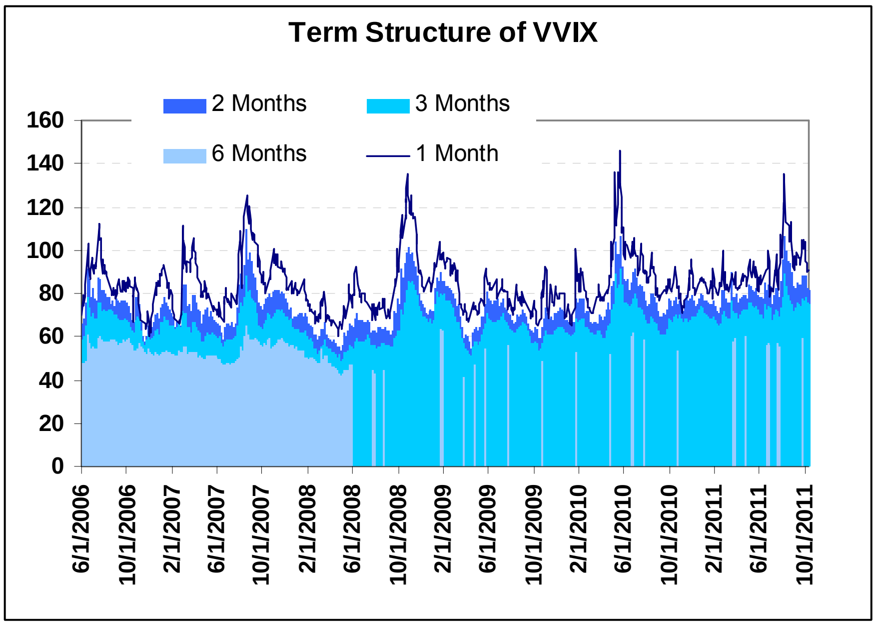
Table 1 Mean Reversion of VVIX

VVIX Range	VVIX 30 Days Later	VVIX 60 Days Later	VVIX 90 Days Later
30 - 50	78	74	71
50 - 60	78	83	82
60 - 70	82	86	87
70 -80	85	85	86
80 - 90	90	86	85
90 - 100	95	90	85
100 - 110	96	89	92
110 - 120	95	92	93
120 - 130	96	94	91
130 - 145	99	93	91

❖ **The term structure of the VVIX is downward sloping**

Based on past history, the term structure of the VVIX is downward sloping. This is normal because a forward price is a forecast, and a short term forecast tends to be more volatile than a long-term forecast.

Chart 2 Term Structure of the VVIX



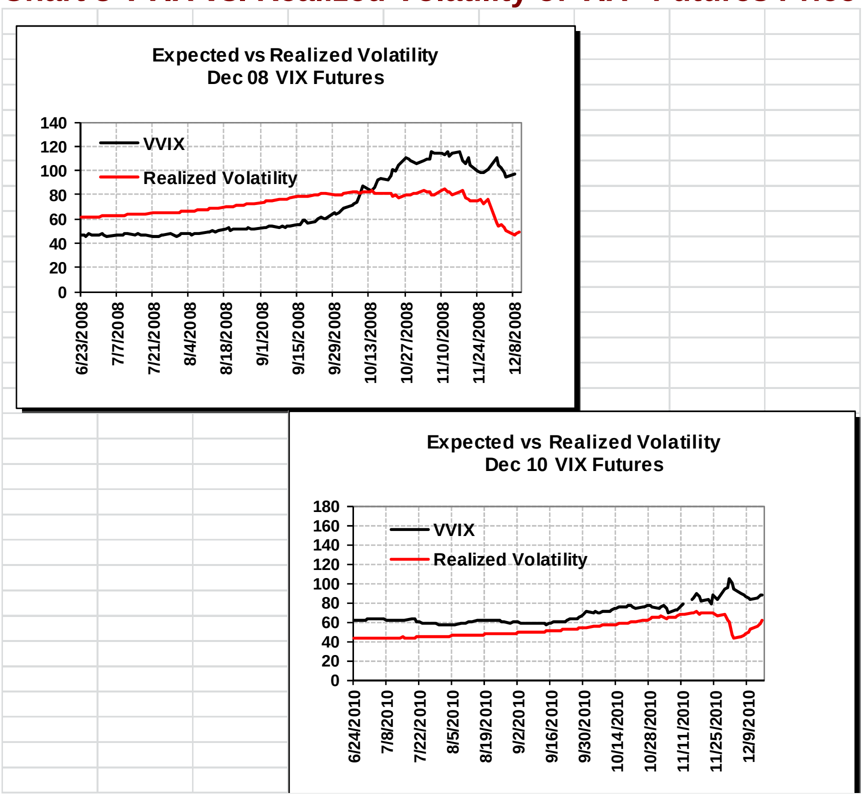
2. Applications of the VVIX

❖ **Volatility Premium Capture**

The VVIX and its term structure price the volatility of VIX® futures prices. This is useful information for trading that volatility. VVIX embeds a risk premium that can be captured by selling the VVIX portfolio on a consistent basis. The risk premium is illustrated in the bottom panel of Chart 3 where there is a positive spread between the daily value of the December 2010 VVIX and the volatility subsequently realized until the December 2010 expiration.

The top panel of Chart 3 shows that the risk premium is offset in periods when the market significantly underestimates future realized volatility, as it did in September and October 2008, the onset of the credit crisis. An investor who sensed that VVIX undervalued future volatility prior to that period could have profited by buying the VVIX portfolio.

Chart 3 VVIX vs. Realized Volatility of VIX® Futures Price



- **Points on the VVIX Term Structure yield estimates of the fair value of the VIX® at expiration dates**

At date t, VVIX $_{t,T}$ is the point on the VVIX term structure corresponding to expiration date T. VVIX $_{t,T}$ generates an alternative estimate of the final settlement value of VIX futures expiring at T. This estimate is an approximate "fair value" for VIX® futures which is based on VIX® option prices.

$$\text{Fair value}_t = E_t[VIX_T] = E_t[F_T] \approx \sqrt{E_t[F_T^2] - F_t^2 * \frac{T-t}{365} VVIX_{t,T}^2}$$

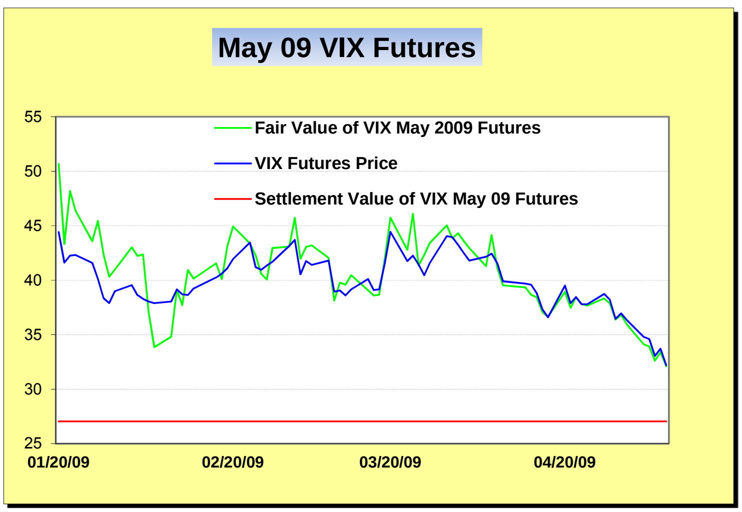
The derivation of the fair value is based on a result of B. Dupire described in Appendix 2.

The first term under the square root sign, $E_t[VIX_T^{\ 2}]$, is the expected variance of the S&P 500 from T to T+30. It is estimated from S&P 500 option (SPX) strips that replicate spot variances to different dates. The second term is a multiple of $VVIX_{t,T}$.

Any deviation between the fair value and the price of VIX futures is a potential signal to buy or sell VIX futures. It also suggests the following quasi-arbitrage: a spread between the portfolio of SPX options that replicates the forward S&P 500 variance VIX_T^2 and a position in VIX® futures that approximately replicates the same forward variance.

The fair value of VIX May 2009 VIX® futures, the VIX® futures price, and the final settlement value are shown in Chart 4.

Chart 4. Fair Value of May 2009 VIX Futures



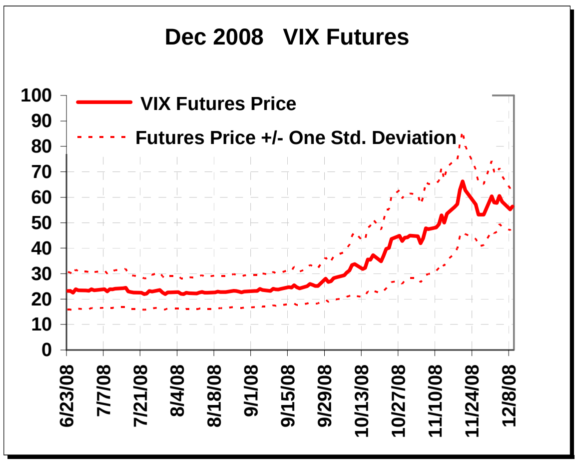
- **Points on the VVIX Term Structure proxy for the standard deviation of the VIX® at expiration dates**

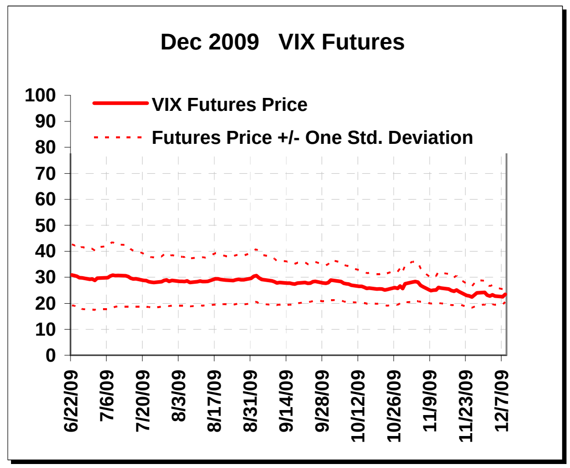
The standard deviation expresses the degree of confidence the market has in forecasts of VIX. For any expiration of VIX® futures, the standard deviation is obtained by multiplying the value of the corresponding VVIX with the price of VIX® futures. VVIX is deannualized and expressed in decimals. The derivation is described in more detail in the Appendix.

Paths that are one standard deviation from the mean are shown in Chart 5 for December 2008 and December 2009 VIX® futures. Uncertainty about the expiration value of December 2008 VIX futures started to increase in

September 2008 – the onset of the credit crisis - and still prevailed close to expiration. The picture for December 2009 is closer to the norm. Uncertainty about the expiration value decreases over the life of the futures contract.

Chart 5. Estimated Standard Deviation of VIX®, Dec 2008 and Dec 2009 VIX®





Appendix 1

Calculation of VVIX

VVIX is calculated with the same methodology as VIX. It is derived from the price of a portfolio of liquid at and out-of-the-money VIX options. That portfolio can be traded to manage the volatility risk of exposures to the VIX® and to harvest the risk premium between the expected and realized volatility of VIX® forward prices VVIX is a 30-day volatility but VIX options expiring in 30 days are usually not available. VVIX-like values are therefore calculated from VIX® options expiring at two dates straddling 30 days. VVIX is then interpolated from these numbers.

At each expiration, VVIX is calculated from VIX® options prices using the VIX® formula:

$$\sigma^2 = \frac{2}{T}\sum_i \frac{\Delta K_i}{K_i^2} e^{RT} Q(K_i) \quad - \quad \frac{1}{T}\left[\frac{F}{K_0} - 1\right]^2 ,$$

where…

σ is	$VVIX/100 \Rightarrow$ VVIX = $\sigma \times 100$
T	Time to expiration
F	Forward index level derived from index option prices
K_0	First strike below the forward index level, F
K_i	Strike price of i^{th} out-of-the-money option; a call if K_i>K_0 and a put if K_i< K_0; both put and call if K_i=K_0.
ΔK_i	Interval between strike prices – half the difference between the strike on either side of K_i:

$$\Delta K_i = \frac{K_{i+1} - K_{i-1}}{2}$$

(*Note*: ΔK for the lowest strike is simply the difference between the lowest strike and the next higher strike.

Likewise, ΔK for the highest strike is the difference between the highest strike and the next lower strike.)

R	Risk-free interest rate to expiration
$Q(K_i)$	The midpoint of the bid-ask spread for each option with strike K_i.

Since the step-by-step calculation of VVIX follows that of the VIX, please refer to the VIX White Paper for additional details

http://www.cboe.com/micro/VIX/VIXwhite.pdf

Note that VIX options settle on Wednesdays that precede SPX expirations by 30 days.

Appendix 2

1. Estimation of Standard Deviation of the VIX® from VVIX

At date t and for expiration date T,

$$VVIX_{t,T}^{2} = \frac{365}{T-t} * E_t[\int_t^T \sigma_{R_{tt}}^2 \, dt] \approx \frac{365}{T-t} * VAR_t[\ln(\frac{F_T}{F_t})] = \frac{365}{T-t} * VAR_t[\ln(F_T)]$$

where R_t is the logarithm of the instantaneous return of the forward price F_t and $F_T = VIX_T$.

A truncated Taylor expansion of the logarithm of F_T is

$$\ln(F_T) \approx \ln(F_t) + \frac{F_T - F_t}{F_t}$$

This implies

$$VAR_t[\ln(F_T)] \approx \frac{VAR_t[F_T]}{F_t^2} \mapsto VAR_t[F_T] = VAR_t[VIX_T] \approx F_t^2 * \frac{T-t}{365} VVIX_{t,T}^2$$

$$VAR_t[VIX_T] \approx F_t^2 * \frac{T-t}{365} VVIX_{t,T}^2$$

Taking square roots on both sides, the standard deviation of VIX_T is linked to $VVIX_{t,T}$.

$$STDV_t[VIX_T] \approx F_t * \sqrt{\frac{T-t}{365}} VVIX_{t,T}$$

2. Fair Value of VIX Futures

The fair value of VIX® futures prices is

$$\text{Fair value}_t = E_t[VIX_T] = E_t[F_T]$$

This fair value is derived by B. Dupire In "Volatility Derivatives Modeling" (Bloomberg Presentation, 2005) by expanding the variance of the settlement price of VIX futures as

$VAR_t[F_T] = E_t[F_T^{\,2}] - (E_t[F_T])^2 \rightarrow$ Fair value$_t$ = $\sqrt{E_t[VIX_T^2] - VAR_t[VIX_T]}$

The fair value of VIX_T is equal to the square root of the difference between the date T forward value of VIX^2 and the variance of F_T (=VIX_T). From section 1 above, we can approximate this variance as:

$$VAR_t[VIX_T] \approx F_t^2 * \frac{T-t}{365} VVIX_{t,T}^2$$

3. Fair Value Strategy

Suppose that the fair value of VIX_T is greater than the current price of VIX® futures.

$$\sqrt{E_t[VAR_{T,T+30]} - c} > F_t$$

where c is $VAR_t[VIX_T]$ which can be estimated from VVIX.

Then

$$E_t[VAR_{T,T+30]} - c > F_t^2$$

This payoff is replicated with a bond and a position in SPX option strips that pays off the 30-day variance of the S&P 500 from T to T+ 30