Understanding volatility of volatility
CBOE Risk Management Conference Europe 2014

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Motivation
Why do we need care about Vol of Vol (VoV)?

- **VIX options**: robust YoY growth in terms of contracts outstanding & traded volumes

- **Call OI > 3x Put OI in 2014** (vs 1.6x in 2011); potentially explains fast vol crushes
How meaningful is the growth in VIX options vs S&P 500 options for vol trading?

- While VIX futures dominate vega traded by VIX derivs, VIX option vega has been increasing steadily.

- **VIX option vega traded ~1/6th of S&P500 (& SPY) options:** That’s $21mn vega (per VIX pt) per day.
Other vol of vol markets are growing too: VXX & V2X

- US has dominated in terms of the success of product innovation -> VIX derivatives and more recently, VXX options (> 250% growth from 2013)

- In Europe, V2X trading has increased too (currently V2X options account for ~€400k of vega per day, 1-2% of ESTX50 vega)
Understanding the distribution of volatility
First step to understanding vol of vol is to understand the nature of the distribution of vol

- **No surprises here:** Selling volatility is profitable, the longer you can hold the position

- **Distribution of vol** is unlike that of traditional assets, and this is important to understand before trading vol of vol
What about the level of vol? Does the distribution of returns depend on the vol regime?

<table>
<thead>
<tr>
<th>Low VIX (&lt;= 25&lt;sup&gt;th&lt;/sup&gt; percentile)</th>
<th>High VIX (&gt;= 75&lt;sup&gt;th&lt;/sup&gt; %ile)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency distribution of daily VIX returns when VIX is low (&lt;=25th %ile ~ 15)</td>
<td>Frequency distribution of daily VIX returns when VIX is high (&gt;=75th %ile ~ 24)</td>
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- **Peaked, heavy-tailed distribution when vol is low:** Big % jumps are rare, so media talks up 30% gains in VIX while the overall vol regime is unchanged, parallel with CDS

- **When VIX is high, both left and right tails come into play:** The distribution of returns is less skewed and less peaked

- **When vol is low, vol of vol can be unstable:** Can thinking in terms of vol pt changes help?

Source: BoA Merrill Lynch Global Research, Bloomberg. Data from 3-Jan-90 to 25-Jul-14
What’s more ‘normal’? Both distributions are skewed in low vol & less skewed in high vol periods

In low vol periods distribution of vol pt changes is less peaked (lower kurtosis) than % returns

In high vol periods % returns have less kurtosis => implications for trade sizing, risk management
Volatility of volatility: Realised
What is VoV? Let’s examine realised first

- **VIX Index HVG**: Looks like 100-day realised vol* of VIX averages ~100
- Seems high? As of 28-Aug, 50 delta** Jan-15 VIX call implied vol = 60%... What are we missing?

Source: Bloomberg. *Classical model, e.g. annualised average of squared log returns, **delta as per Bloomberg. Data: 27-Feb-09 to 27-Aug-14
Shorter-dated VIX futures have more beta to VIX than longer-dated (extent of this has increased in latest low vol regime)

As the tenor of a VIX future decreases, it becomes MORE volatile; VIX is most volatile

VXX doesn’t behave in this way, as you always have constant tenor (30-day) exposure to VIX futures
Realised vol of VIX futures: Constant tenor futures help establish bounds; need to calculate walk vol

- Vol of 3M constant tenor future < vol of 2M constant tenor future < vol of 1M constant tenor future

- To calculate realised vol of vol properly, need to “walk down with the VIX future to expiry”, but this happens only once a month with real futures (for daily data, we need to interpolate)
Volatility of volatility: Implied


VIX options are liquid enough for options prices to just be.

Pricing models can take prices as a reference point and calculate the greeks (implied vol, delta etc)
What does a downward sloping VoV term structure imply for decay of VIX options?

- VIX options decay, but slower than equity options all else equal

- The steep time decay at the end also implies a very peaked gamma profile (theta is gamma rent!)
Relating implied VoV to other observables in the derivatives market

Equation alert; but it will be short!

\[
Var(V_{fut_T}) = E\left[V_{fut_T}^2\right] - (E[V_{fut_T}])^2
\]

\[
= E\left[V_{fut_T}^2\right] - V_{fut_{now}}^2
\]

\[
\approx SPX\text{ f}wd\text{ var}_{now} - V_{fut_{now}}^2
\]

- If VIX option prices were not “a given”, VIX option volatility could be linked to observable metrics like:
  - Difference b/w S&P 500 fwd var & VIX futures
  - S&P 500 skew
  - Realised vol of VIX futures

Arguably, these relationships are more important for newer VoV markets, like the V2X
Enter VVIX: vol of vol measure incorporating skew of VIX options

- VVIX is calculated from VIX option strips in the same way VIX is calculated from S&P 500 options

- It is effectively the **fair strike of a variance swap on the VIX!**
Visualising the VoV risk premium

- Median 1M implied to subsequent realised >> average (VIX vol risk premium is fat-tailed)

- Using daily data & interpolation of VIX futures, median 1M spread = 12 vols from Sep-09 to Jul-14
Since 2013, VoV risk premium has exhibited an unusually fat tailed distribution.

3M vol of vol risk premium, however, has been more stable.
**Equities vs Vol vs VoV in Europe & the US**

- **Equities (69% correl)**
  - SX5E
  - SPX (RHS)

- **Equity vol (94% correl, med spr = 4.6)**
  - SX5E 1M ATMf vol
  - SPX 1M ATMf vol

- **Vol of vol (73% correl, med spr = -2.6)**
  - Var of V2X*
  - Var of VIX*

- **SX5E & SPX are almost 70% correlated** to each other (Mar-10 to Aug-14, using daily returns)

- **1M SX5E and SPX vols are more correlated** => more arbitrage potential when vols diverge

- **V2X var is less than VIX var** => VIX call skew is steeper than V2X (more jump risk from low vol)

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Source: BoA Merrill Lynch Global Research, Bloomberg. Data from 22-Mar-10 to 18-Aug-14. *VIX calculation applied to generic 1M V2X and VIX options*
Concluding remarks

- **Vol of vol is a fast growing market;** mainly in the US but also in Europe

- **Understand the distribution of vol**
  - Vol has negative drift
  - Vol % returns are less ‘normally distributed’ in a low vol environment than vol point changes

- **VIX/V2X futures are more volatile near expiry;** leads to slower (initial) decay of VIX/V2X options
  - This makes it easier to “rent” VIX/V2X options as they retain time value better than equity options
  - But higher gamma (particularly at expiry) means greater P&L risk

- **Vol of vol risk premium exists**
  - Implied VoV > subsequent realised VoV
  - Of late the distribution has been unusually fat-tailed, but in the long run, selling vol of vol systematically should be profitable
Questions?
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<table>
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<th>Investment rating</th>
<th>Total return expectation (within 12-month period of date of initial rating)</th>
<th>Ratings dispersion guidelines for coverage cluster*</th>
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<tbody>
<tr>
<td>Buy</td>
<td>≥ 10%</td>
<td>≤ 70%</td>
</tr>
<tr>
<td>Neutral</td>
<td>≥ 0%</td>
<td>≤ 30%</td>
</tr>
<tr>
<td>Underperform</td>
<td>N/A</td>
<td>≥ 20%</td>
</tr>
</tbody>
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