CTC

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The Volatility Surface

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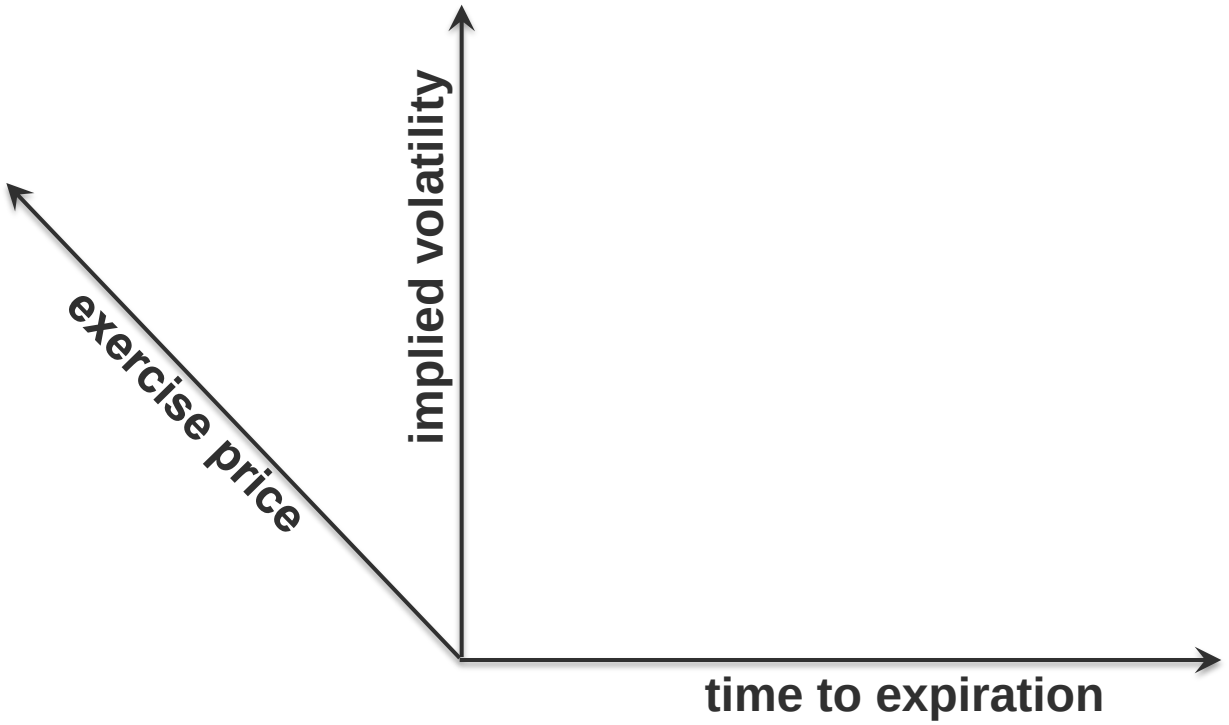
Traders have long noted that implied volatilities vary across both expiration date (the ***term structure*** of volatility) and exercise price (the volatility ***skew***).

When taken together, the term structure and volatility skew form a ***volatility surface***.



Some important questions:

1. Why does the implied volatility vary across expiration dates and exercise prices?

2. How can we model the volatility term structure and skew?

3. How do changes in the term structure and skew affect the risk of an option position?

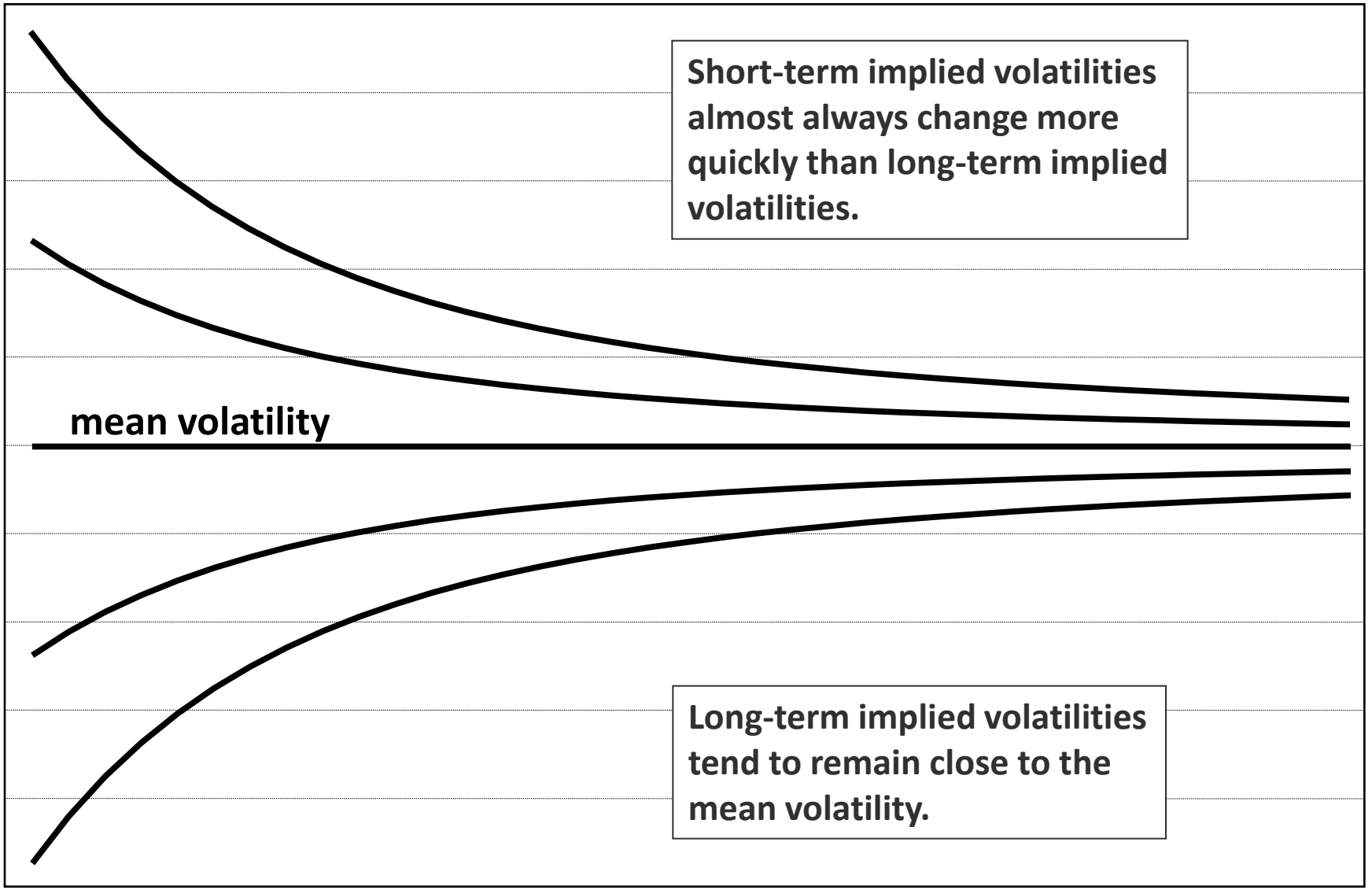
4. Are there trading strategies which focus on changes in the term structure and skew?

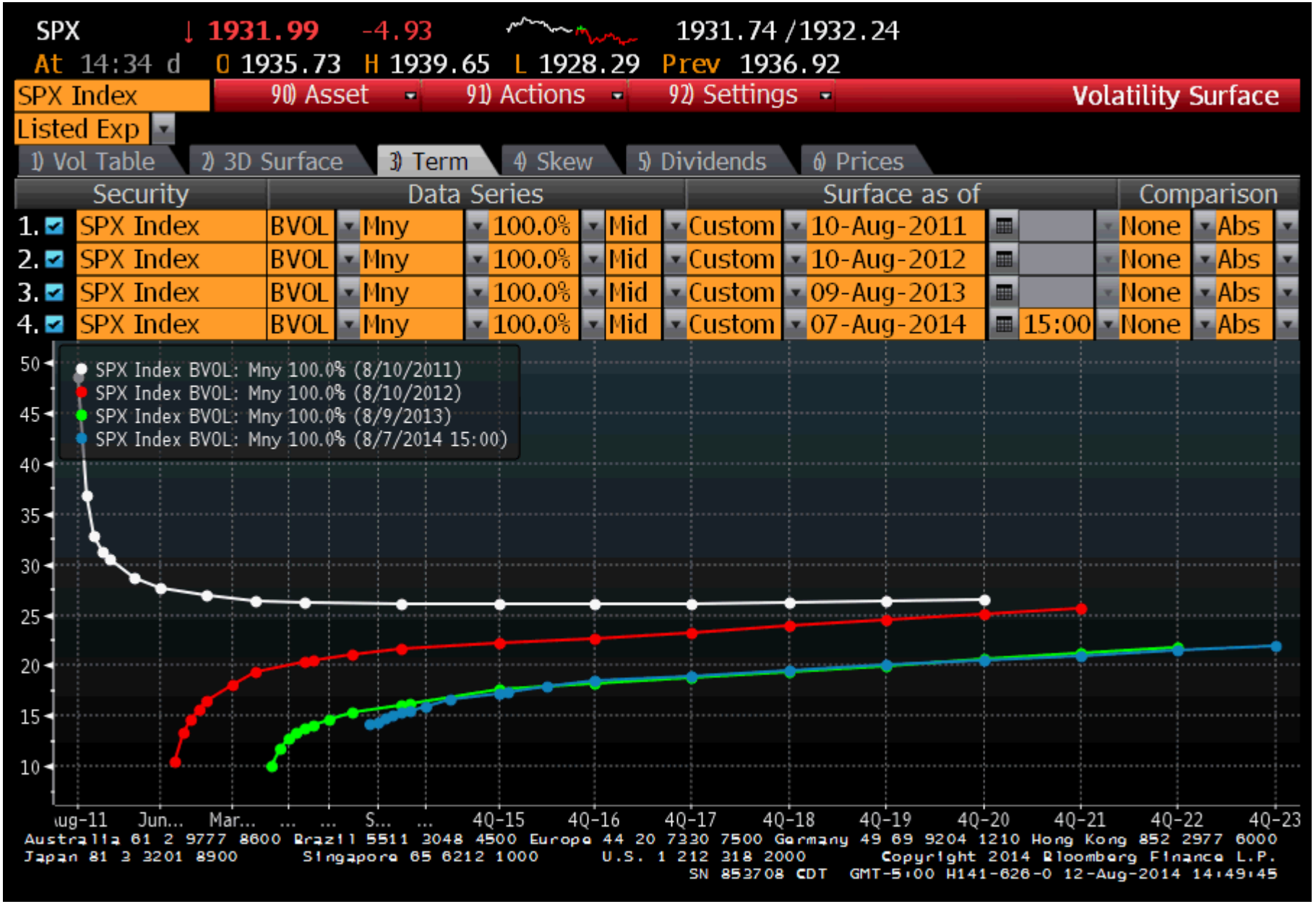
Most underlying contracts have a typical or average volatility to which they tend to revert over long periods of time.

	current implied volatility	volatility rises	volatility falls
March options	25%	30%	20%
June options	25%	28%	22%
September options	25%	26%	24%

Volatility tends to be ***mean reverting***.

Term Structure of Volatility




SPX ↓ 1931.99 -4.93 1931.74 /1932.24
At 14:34 d O 1935.73 H 1939.65 L 1928.29 Prev 1936.92
SPX Index 90) Asset 91) Actions 92) Settings Volatility Surface
Listed Exp
1) Vol Table 2) 3D Surface 3) Term 4) Skew 5) Dividends 6) Prices
Security Data Series Surface as of Comparison
1. SPX Index BVOL Mny 100.0% Mid Custom 10-Aug-2011 None Abs
2. SPX Index BVOL Mny 100.0% Mid Custom 10-Aug-2012 None Abs
3. SPX Index BVOL Mny 100.0% Mid Custom 09-Aug-2013 None Abs
4. SPX Index BVOL Mny 100.0% Mid Custom 07-Aug-2014 15:00 None Abs
SPX Index BVOL: Mny 100.0% (8/10/2011)
SPX Index BVOL: Mny 100.0% (8/10/2012)
SPX Index BVOL: Mny 100.0% (8/9/2013)
SPX Index BVOL: Mny 100.0% (8/7/2014 15:00)
50
45
40
35
30
25
20
15
10
ug-11 Jun... Mar... ... ... S... ... 4Q-15 4Q-16 4Q-17 4Q-18 4Q-19 4Q-20 4Q-21 4Q-22 4Q-23
Australia 61 2 9777 8600 Brazil 5511 3048 4500 Europe 44 20 7330 7500 Germany 49 69 9204 1210 Hong Kong 852 2977 6000
Japan 81 3 3201 8900 Singapore 65 6212 1000 U.S. 1 212 318 2000 Copyright 2014 Bloomberg Finance L.P.
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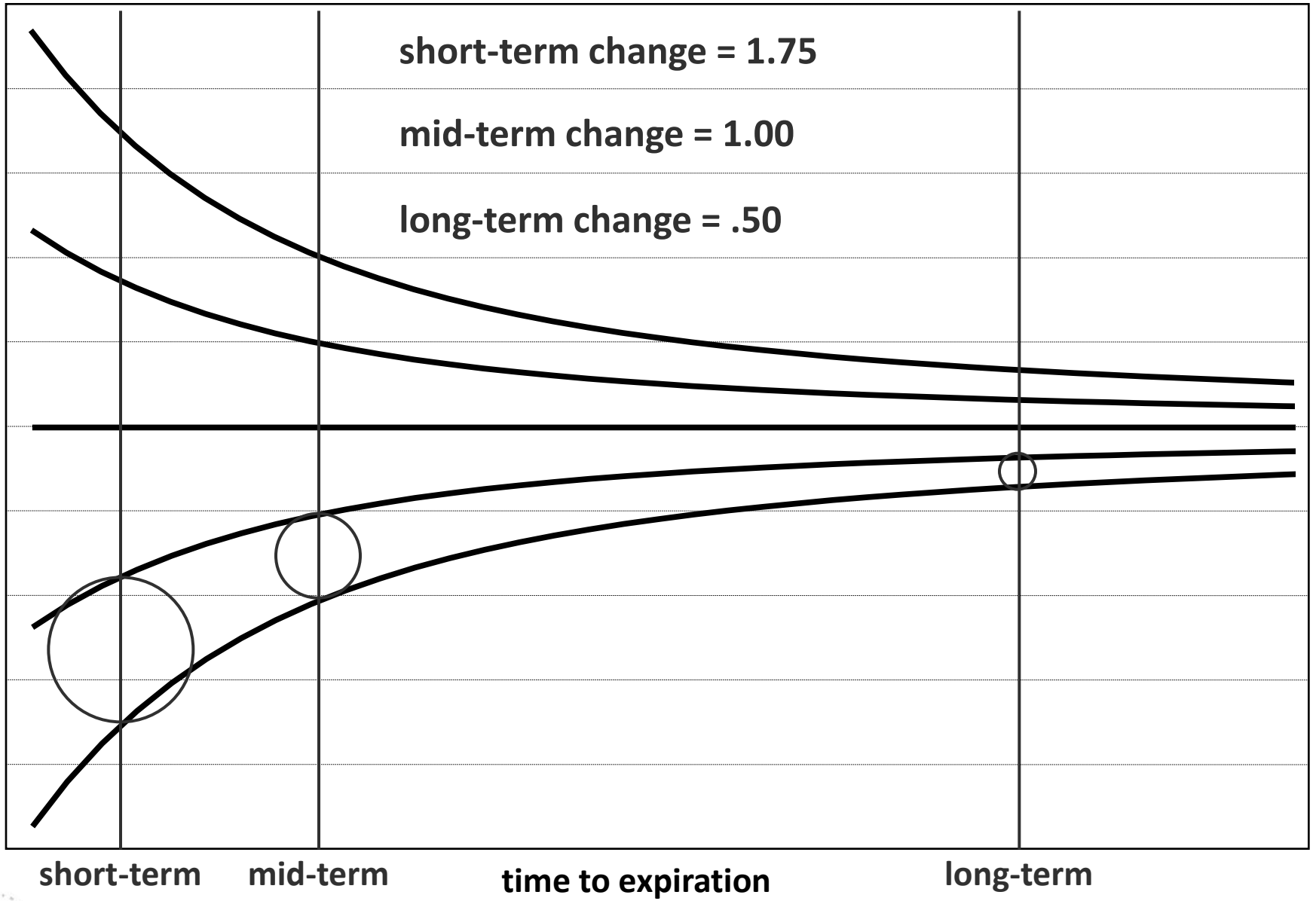
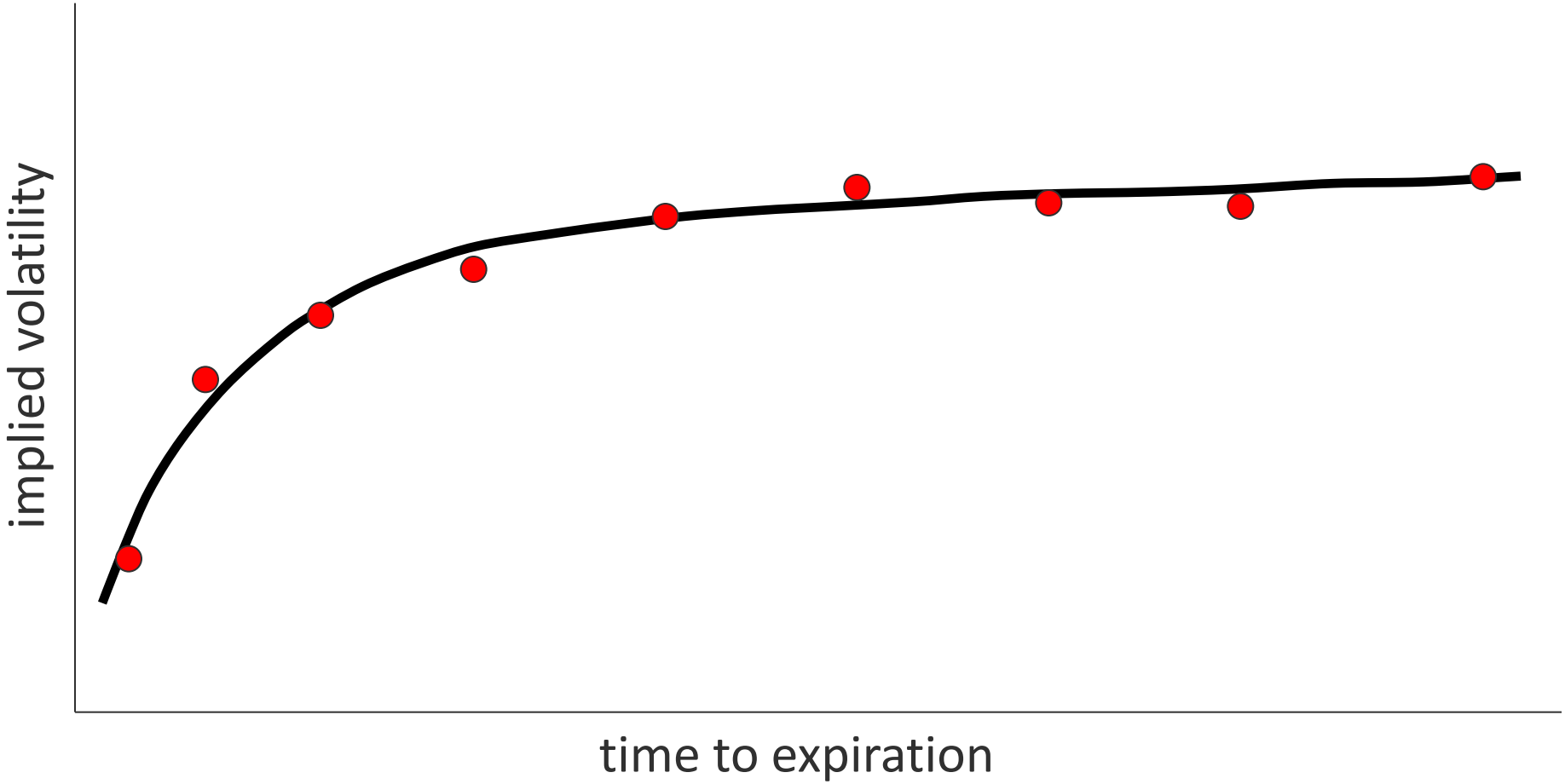
Term Structure of Volatility



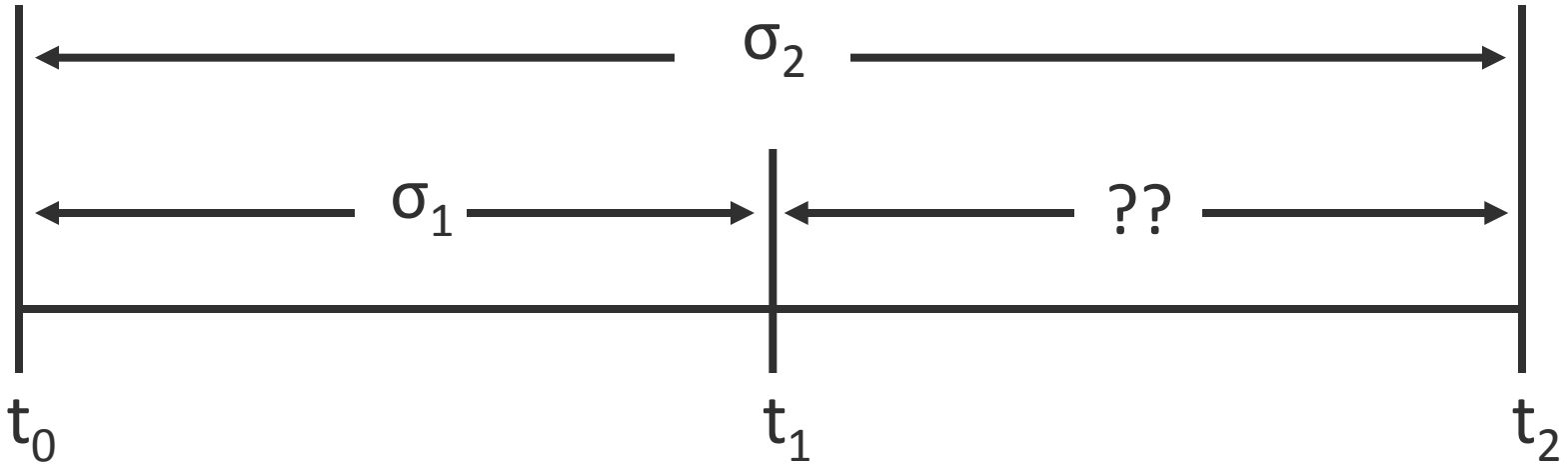
	vega	adjustment	adjusted vega
March	-4.20	1.75	-7.35
June	+2.80	1.00	+2.80
September	+1.40	.50	+.70
total	0	***total adjusted vega***	-3.85

What strategies might make sense?



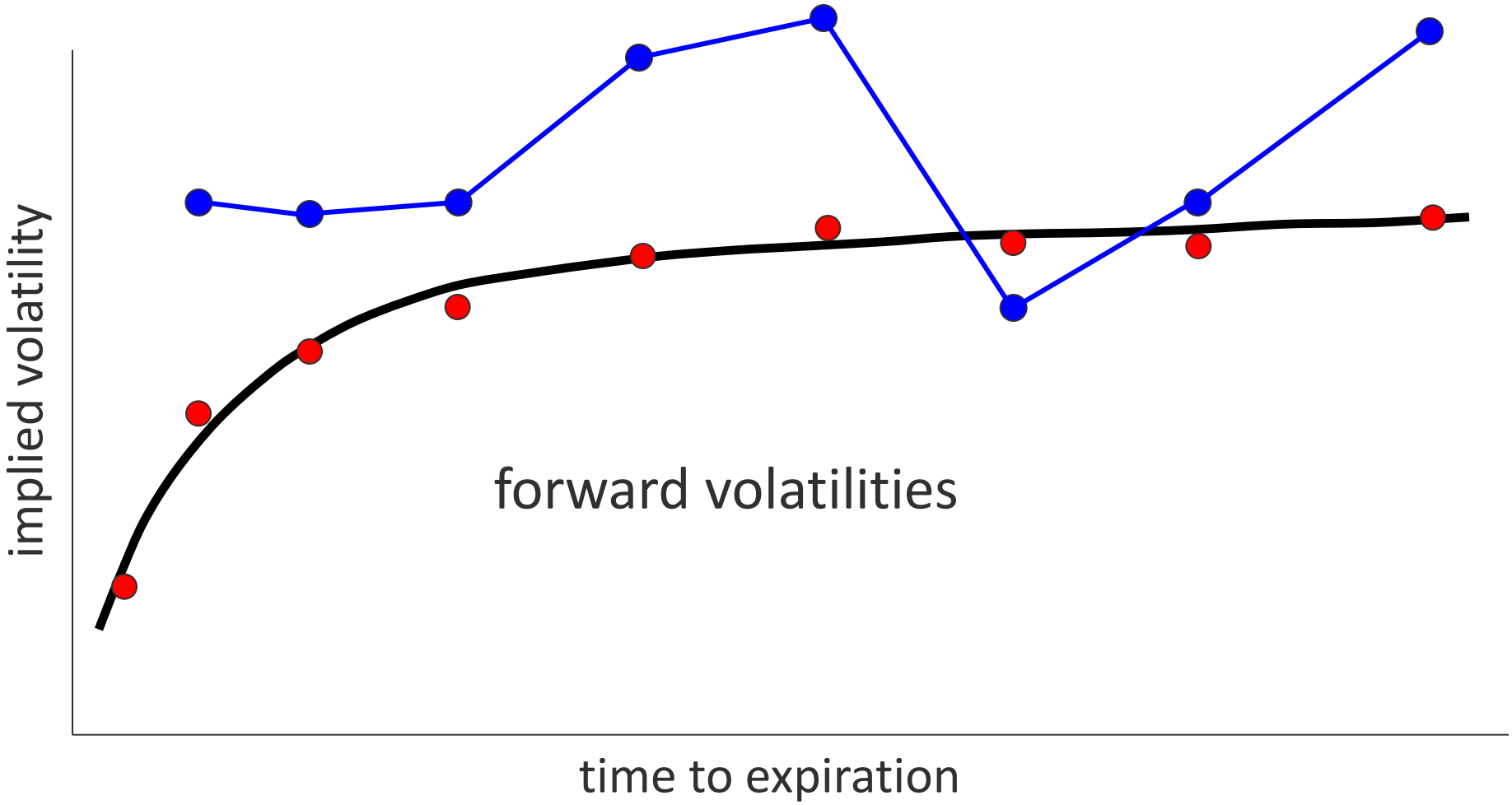
σ_1 = short-term implied volatility

σ_2 = long-term implied volatility

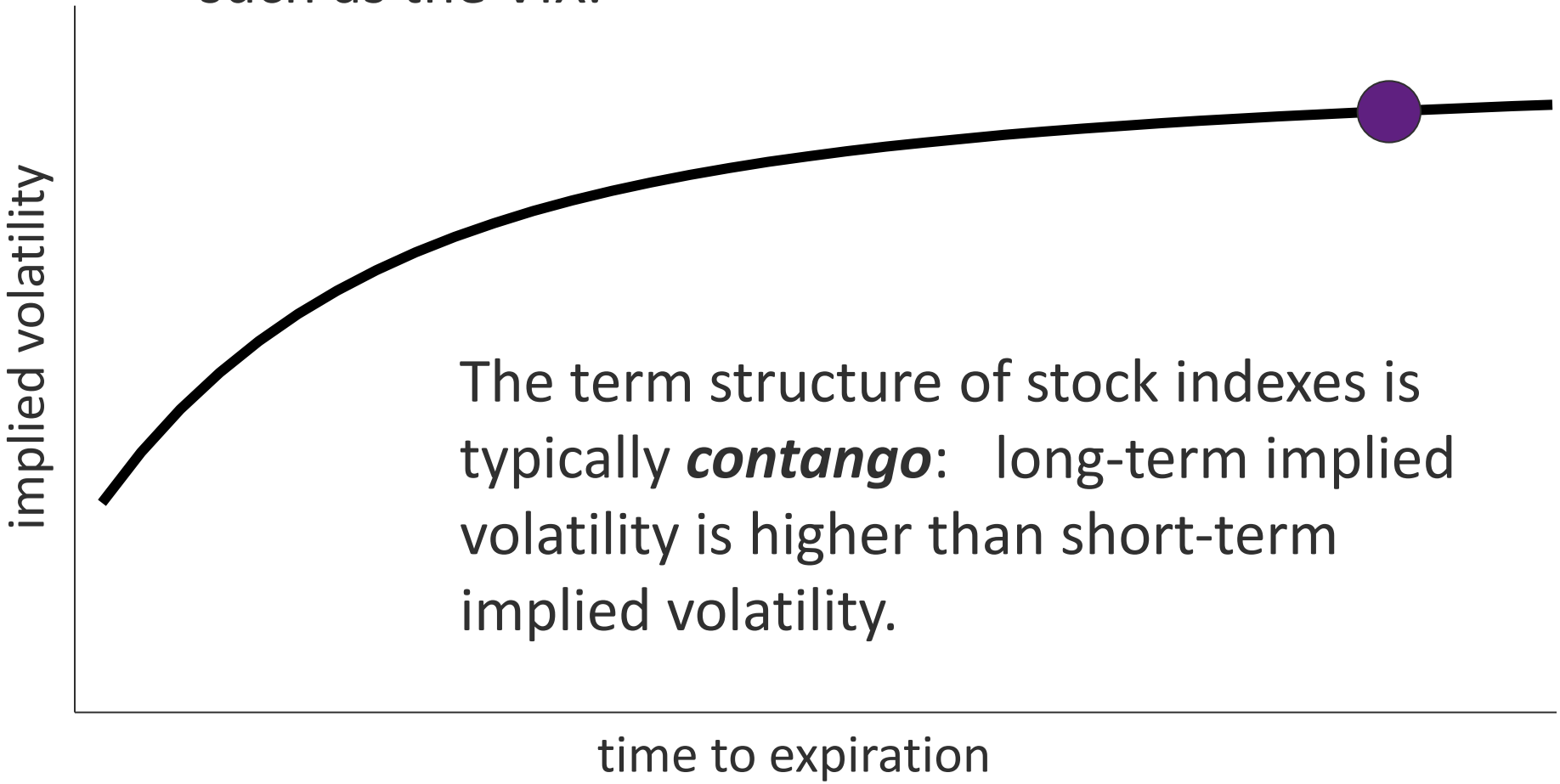


forward volatility $= \sqrt{[(\sigma_2^2 * t_2) - (\sigma_1^2 * t_1)] / (t_2 - t_1)}$

What strategies might make sense?

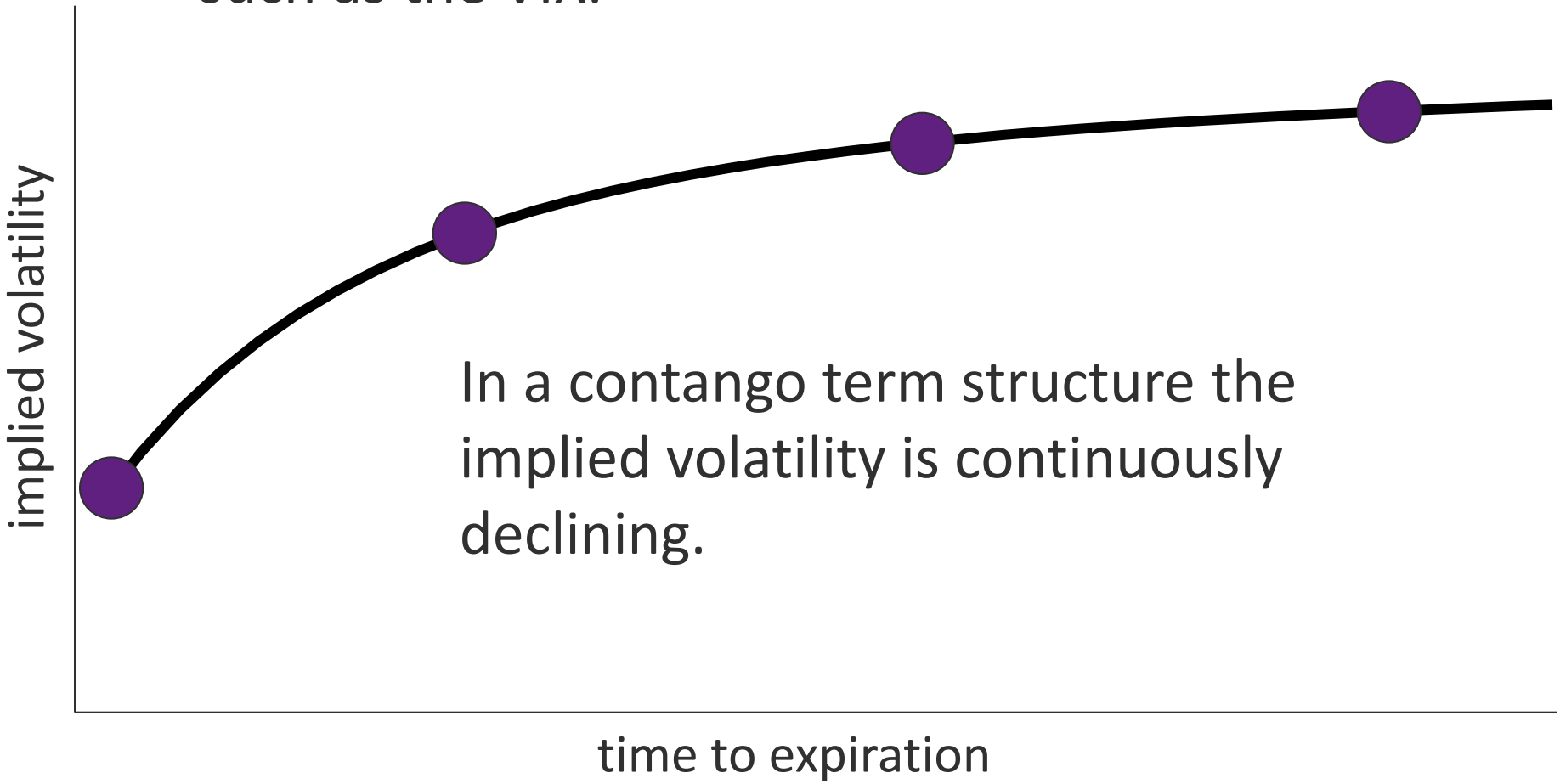


The term structure of volatility has important implications for implied volatility products such as the VIX.



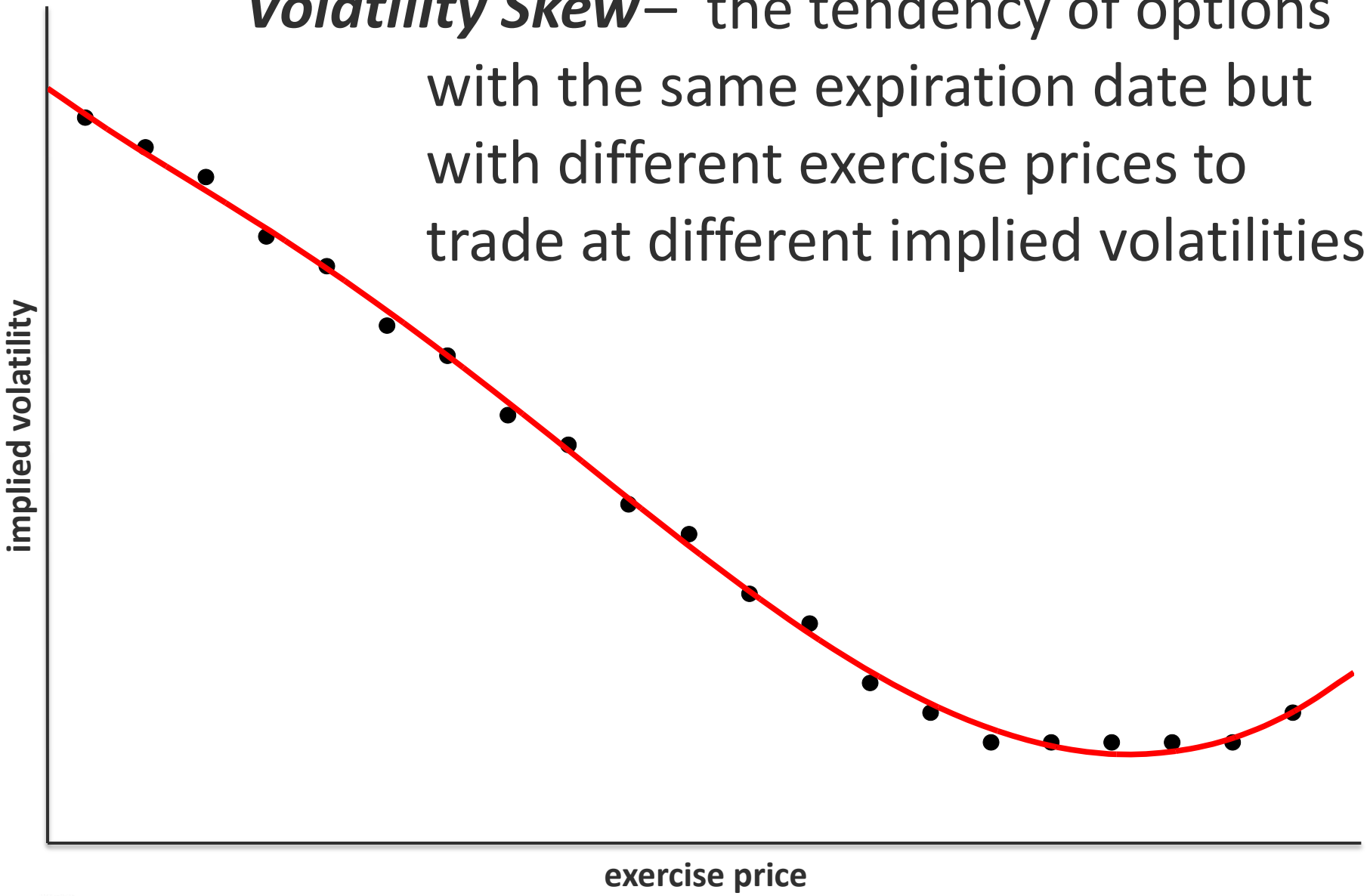
The term structure of stock indexes is typically ***contango***: long-term implied volatility is higher than short-term implied volatility.

The term structure of volatility has important implications for implied volatility products such as the VIX.



In a contango term structure the implied volatility is continuously declining.

Volatility Skew– the tendency of options with the same expiration date but with different exercise prices to trade at different implied volatilities



Long stock
Stock price = 100

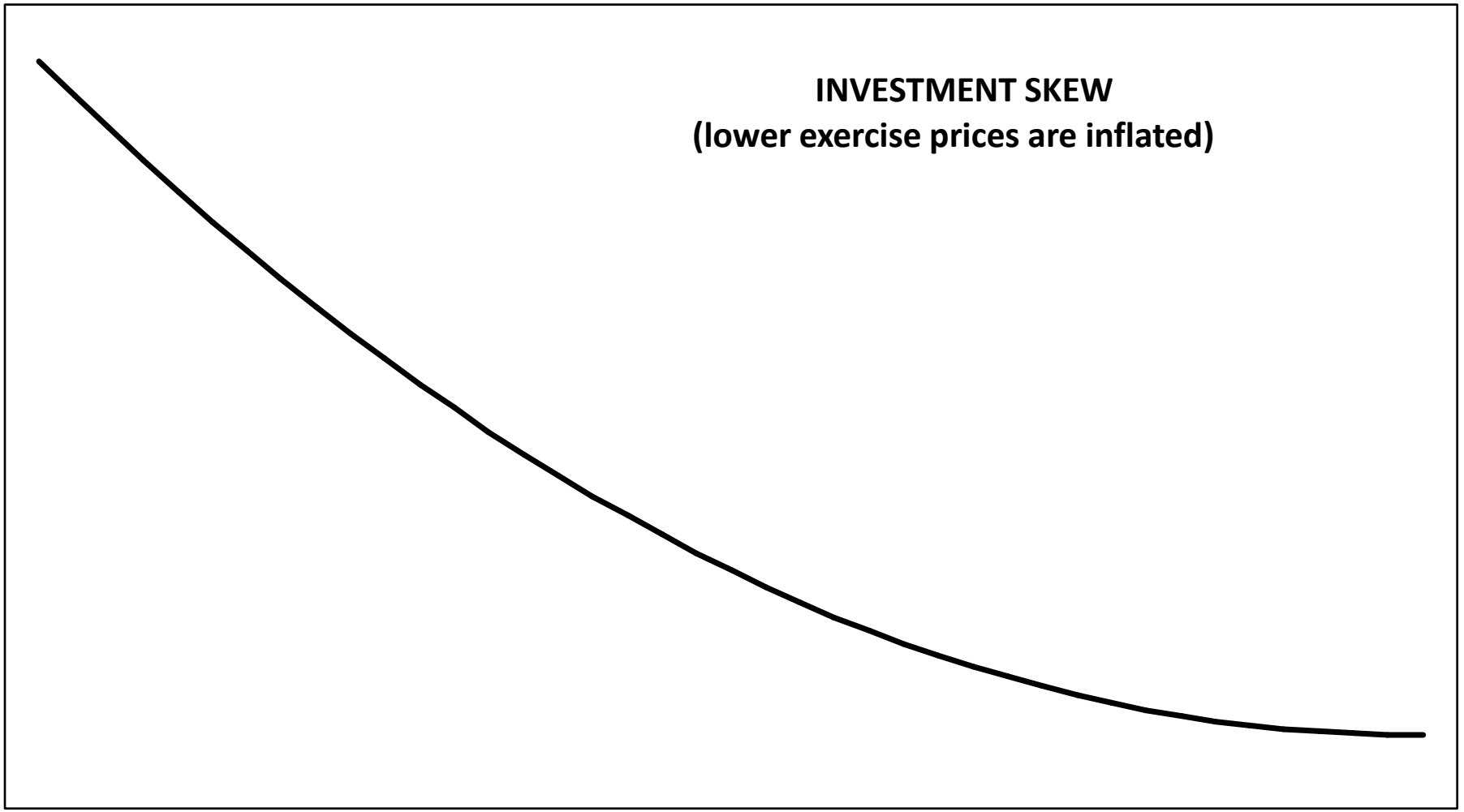
Typical hedging strategies:

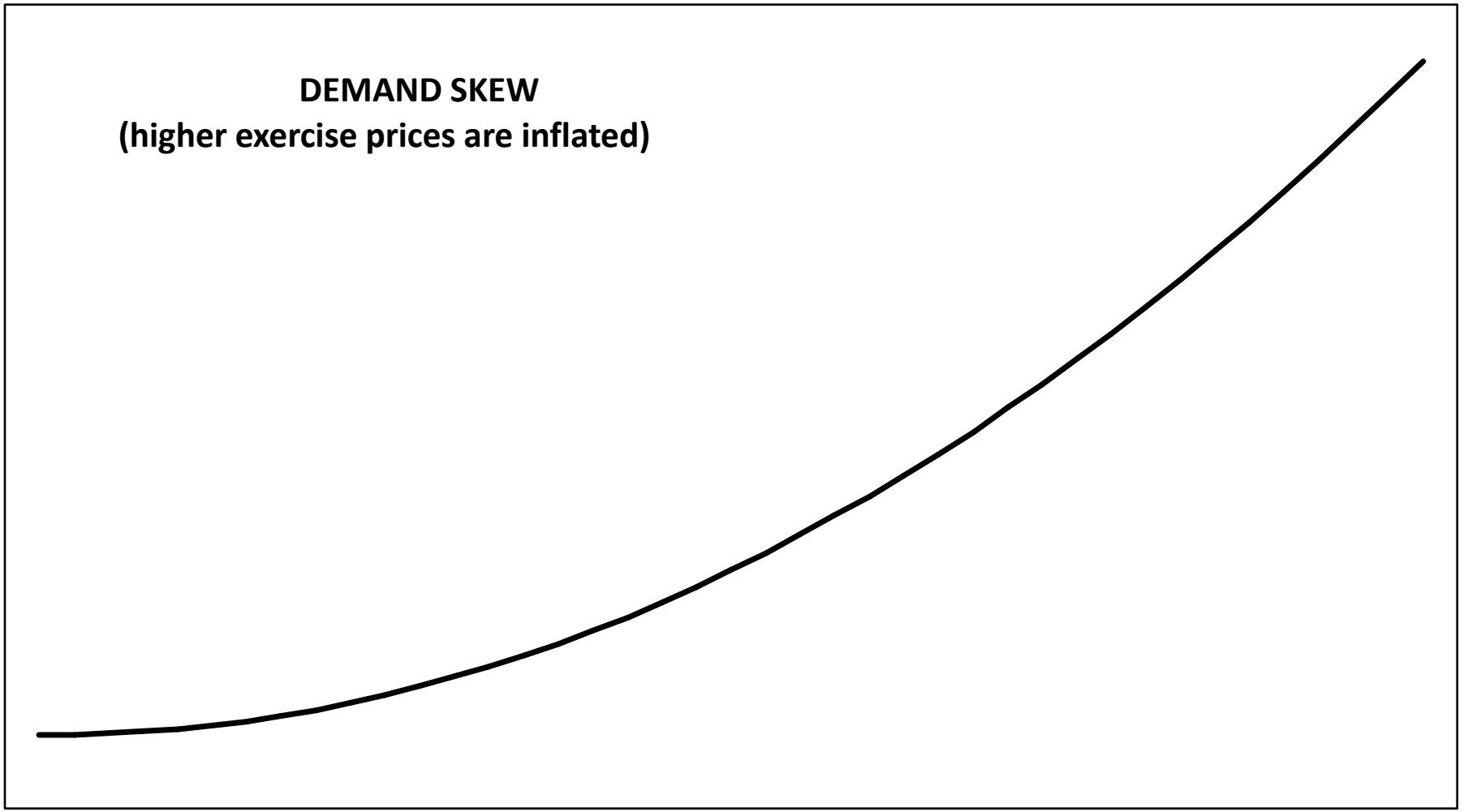
buy protective puts

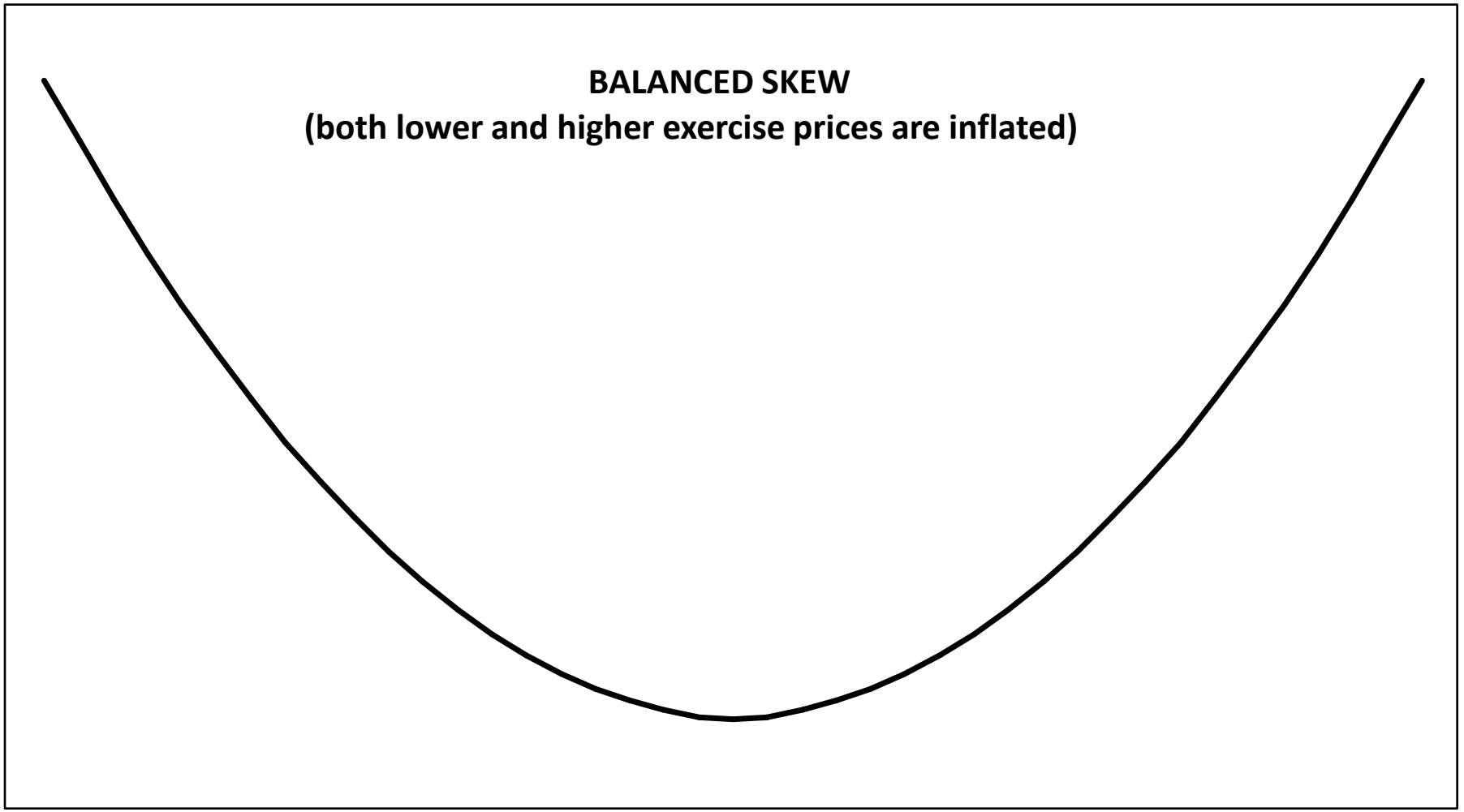
which exercise price 95 / 105 ?

sell covered calls

which exercise price 95 / 105 ?


INVESTMENT SKEW
(lower exercise prices are inflated)
implied volatility
exercise price


DEMAND SKEW
(higher exercise prices are inflated)
implied volatility
exercise price


BALANCED SKEW
(both lower and higher exercise prices are inflated)
implied volatility
exercise price

Some assumptions on which traditional theoretical pricing models are built:

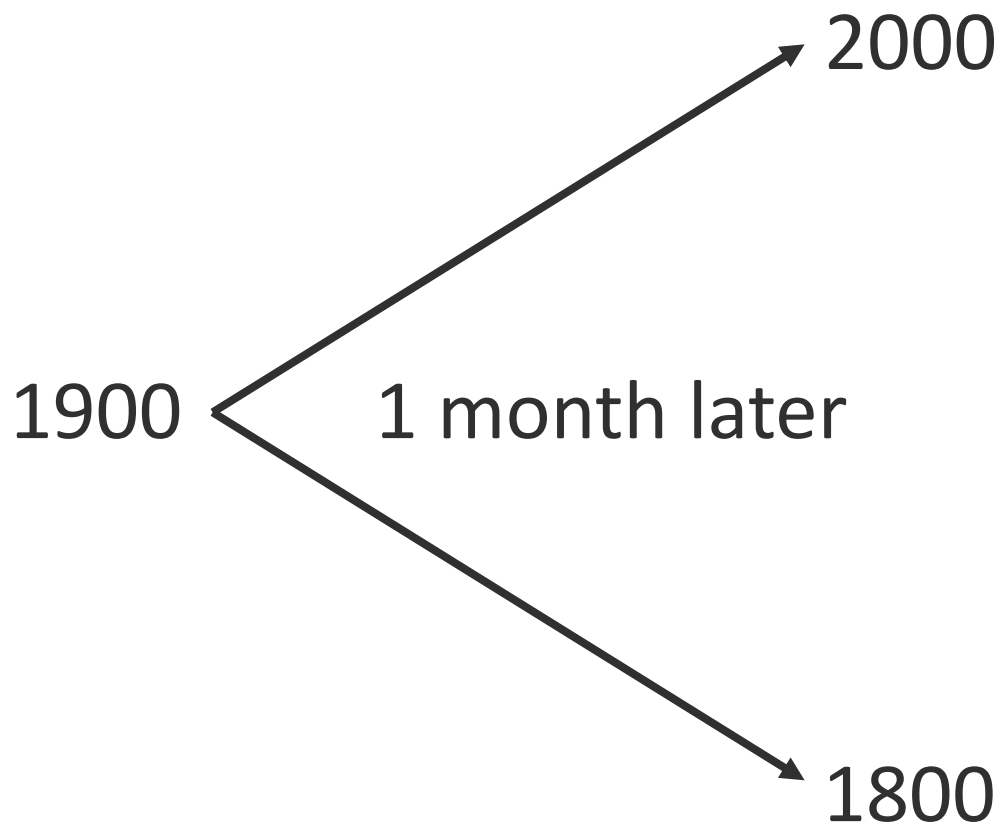
Price changes are normally distributed (prices at expiration are lognormally distributed)

Volatility is constant over the life of the option

Trading is continuous, with no gaps in the price of the underlying contract

Volatility is independent of the direction in which the underlying contract moves

Stock index = 1900



Will the index be more volatile at 1800 or 2000?

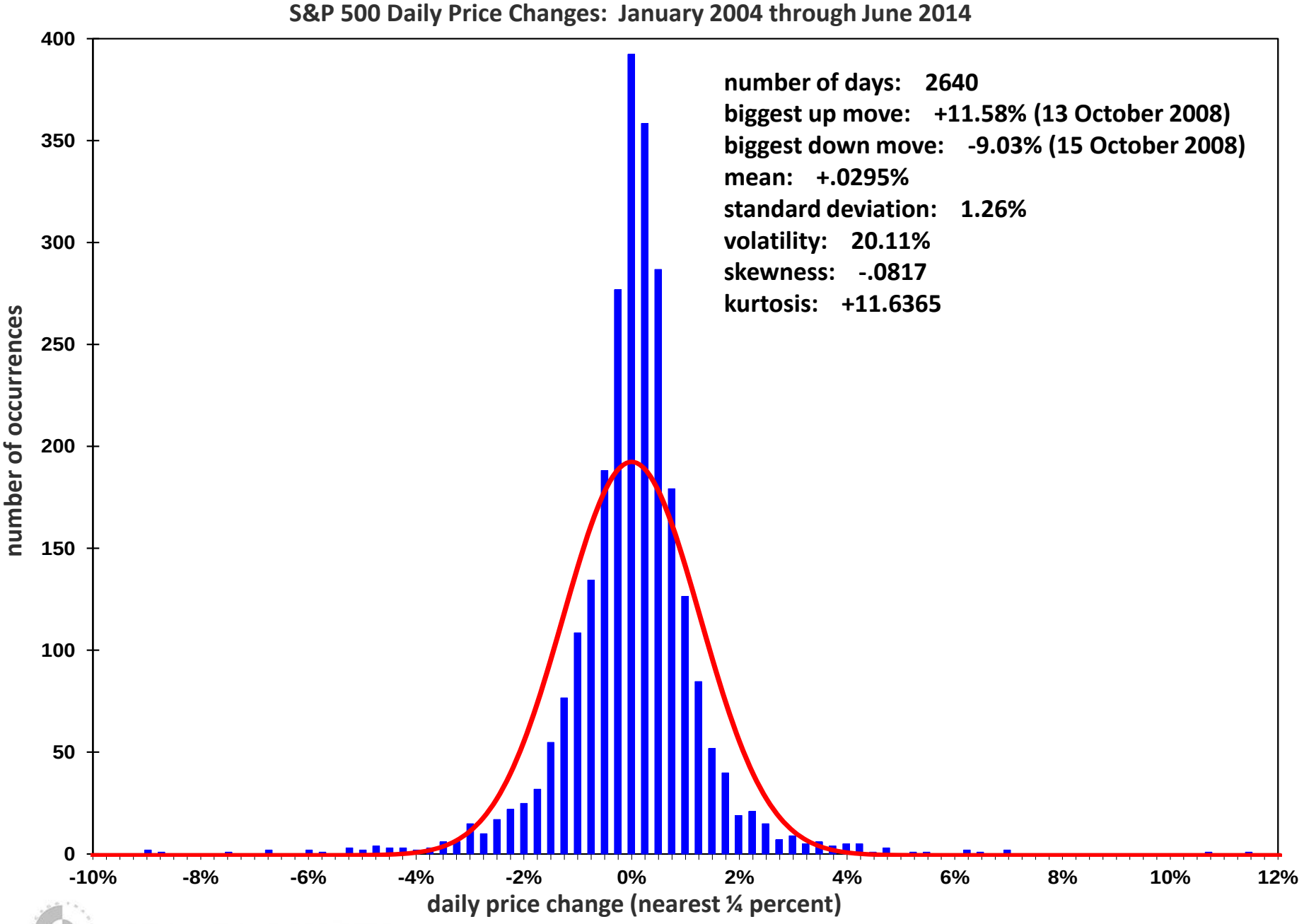
You buy an at-the-money straddle in a stock index.

What is your approximate delta position?

What does ***delta neutral*** mean?

Do you have a preference whether the market rises or falls?

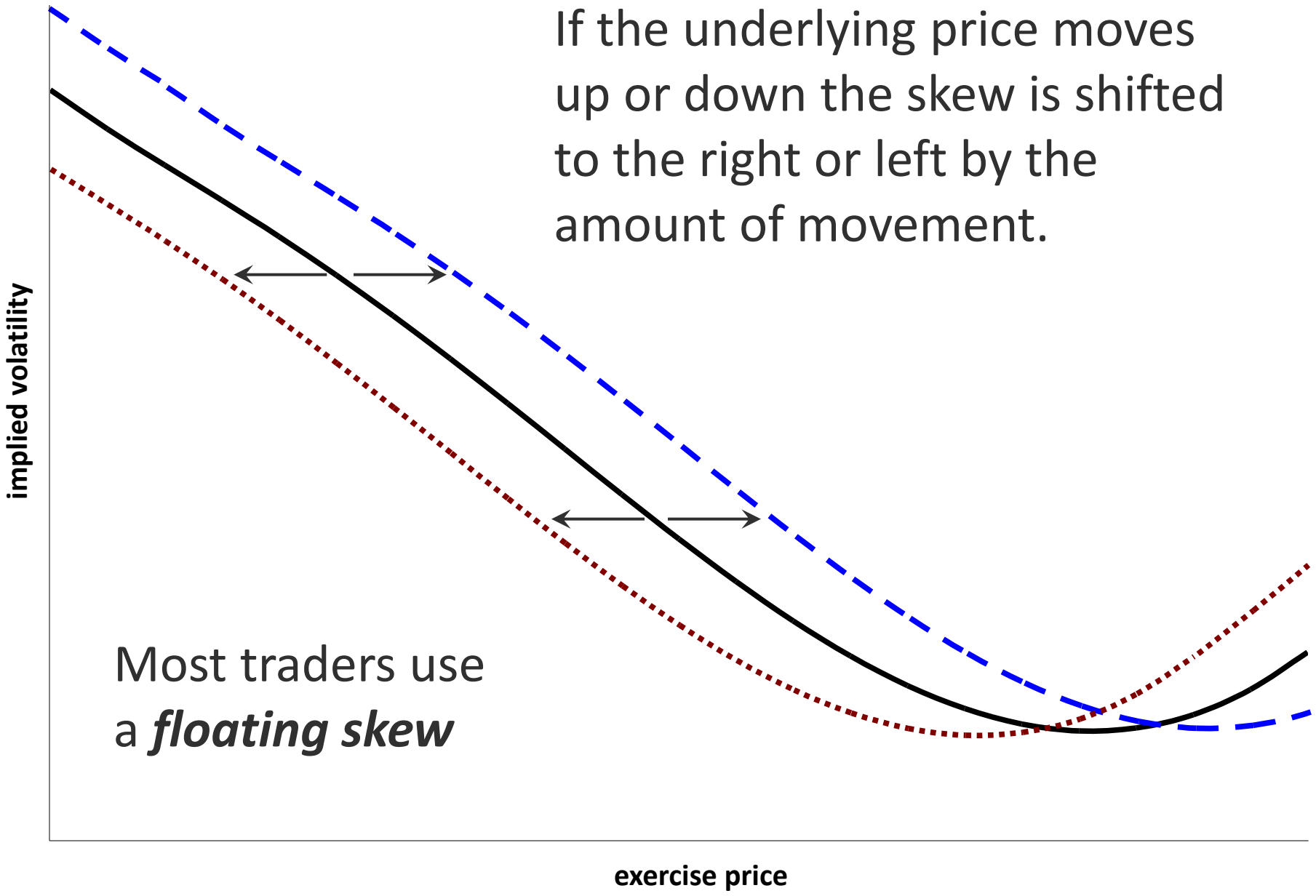
Will it make any difference if the underlying is a physical commodity?

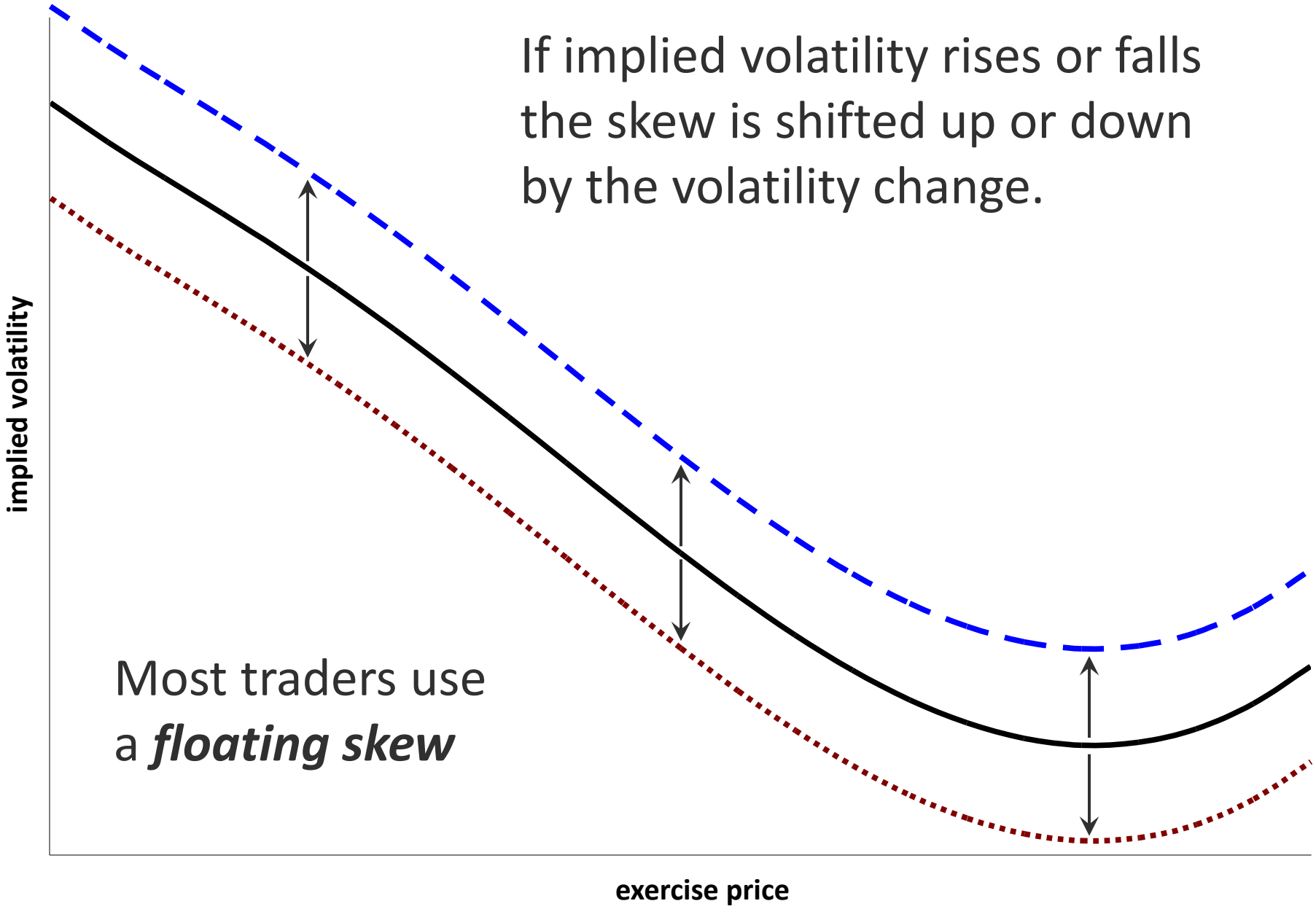

S&P 500 Daily Price Changes: January 2004 through June 2014
number of days: 2640
biggest up move: +11.58% (13 October 2008)
biggest down move: -9.03% (15 October 2008)
mean: +.0295%
standard deviation: 1.26%
volatility: 20.11%
skewness: -.0817
kurtosis: +11.6365
number of occurrences
0
50
100
150
200
250
300
350
400
-10%
-8%
-6%
-4%
-2%
0%
2%
4%
6%
8%
10%
12%
daily price change (nearest ¼ percent)

In order to model risk a trader will want to consider how changes in market conditions are likely to affect…

the location of the skew

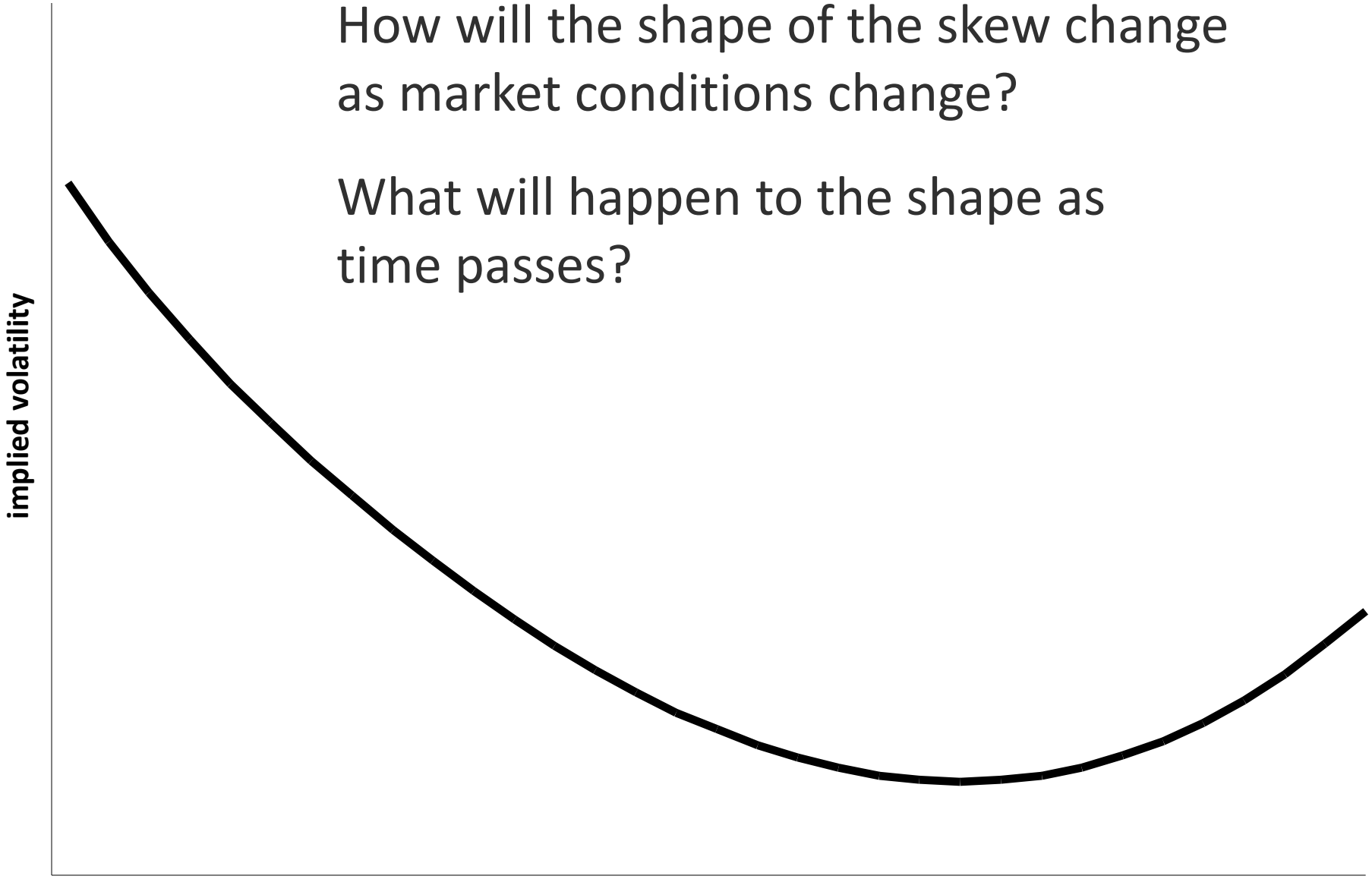
the shape of the skew


If the underlying price moves up or down the skew is shifted to the right or left by the amount of movement.
Most traders use a ***floating skew***
implied volatility
exercise price


If implied volatility rises or falls the skew is shifted up or down by the volatility change.
implied volatility
Most traders use a ***floating skew***
exercise price

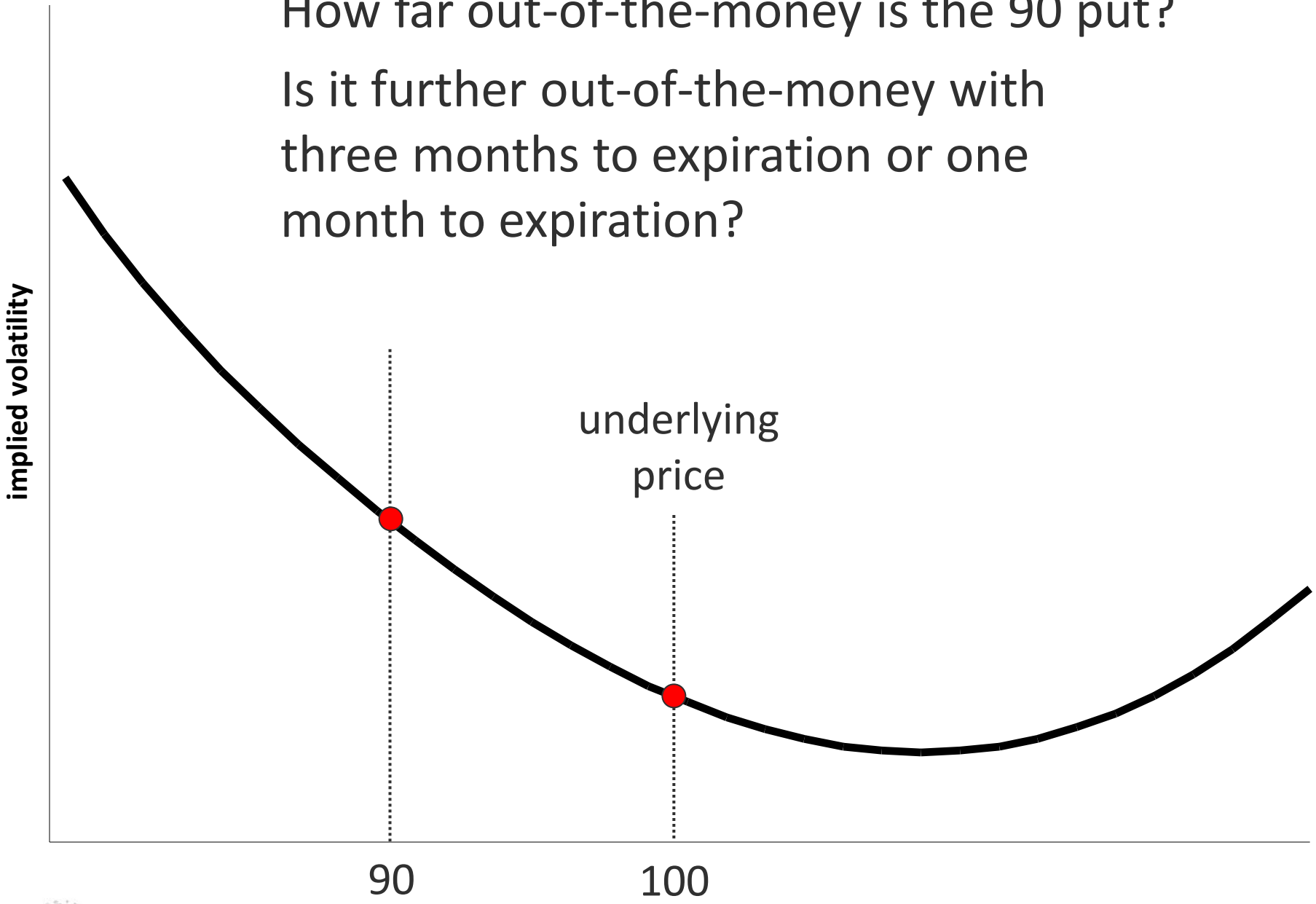
How will the shape of the skew change as market conditions change?

What will happen to the shape as time passes?



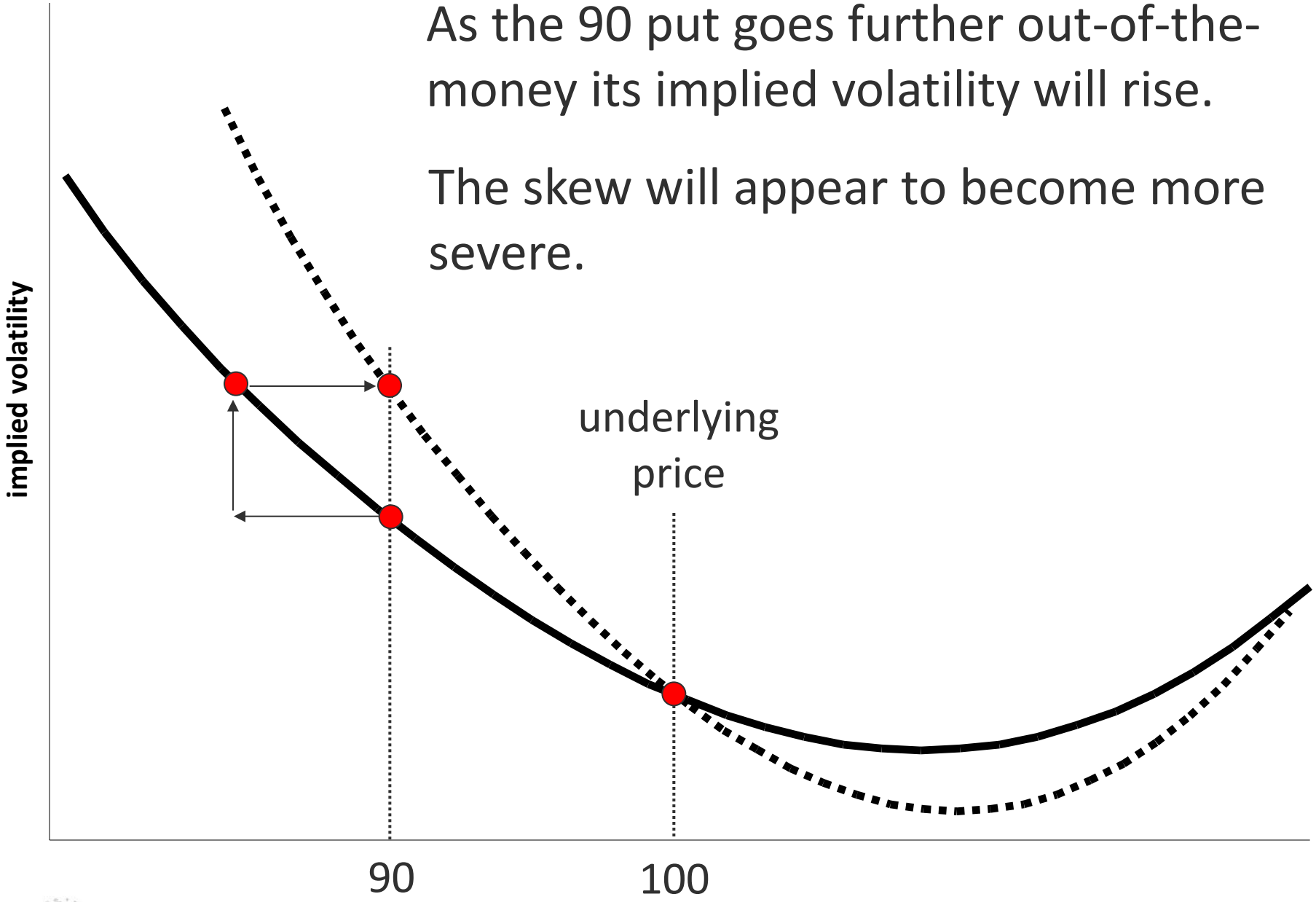
How far out-of-the-money is the 90 put?

Is it further out-of-the-money with three months to expiration or one month to expiration?

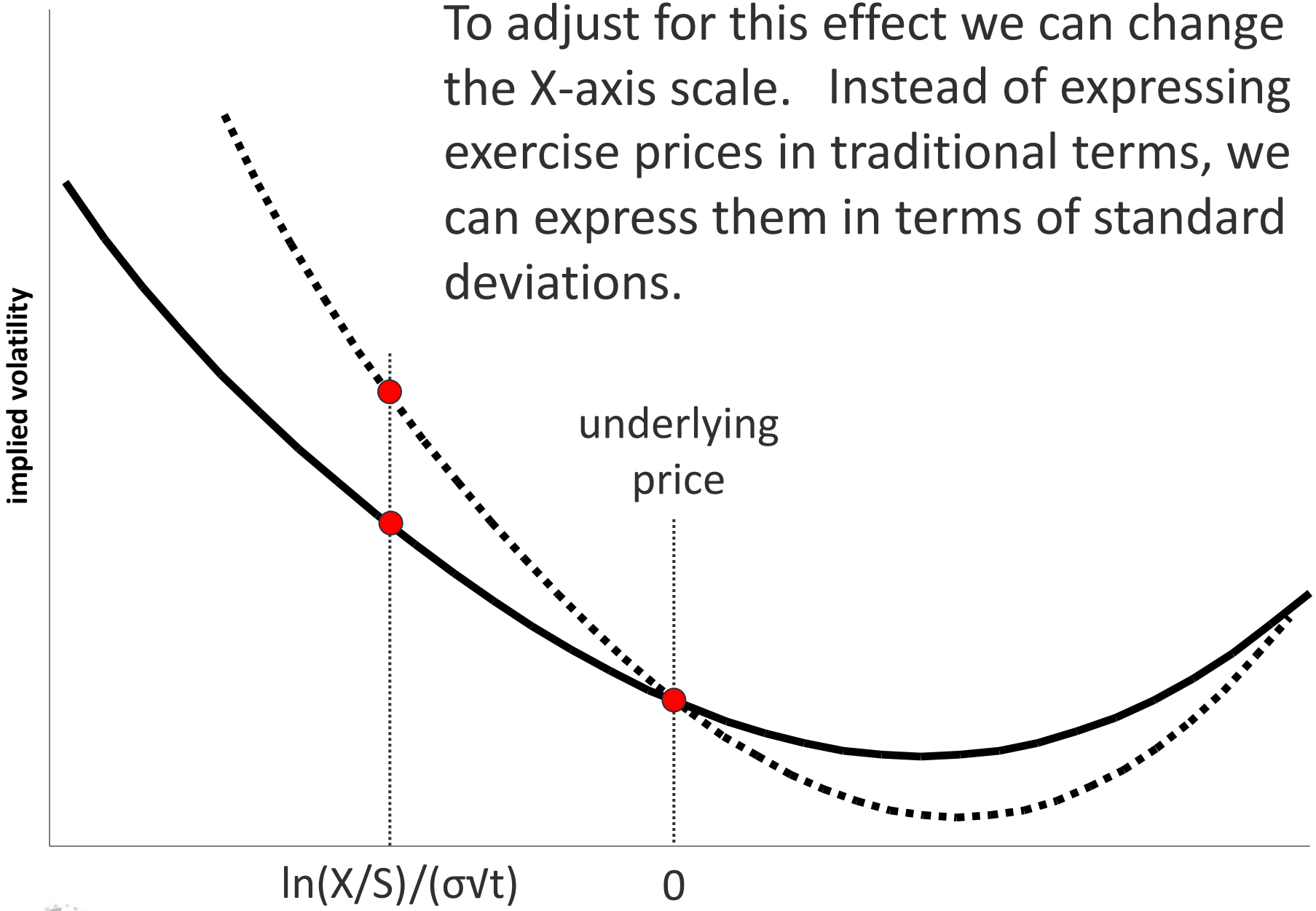


As the 90 put goes further out-of-the-money its implied volatility will rise.

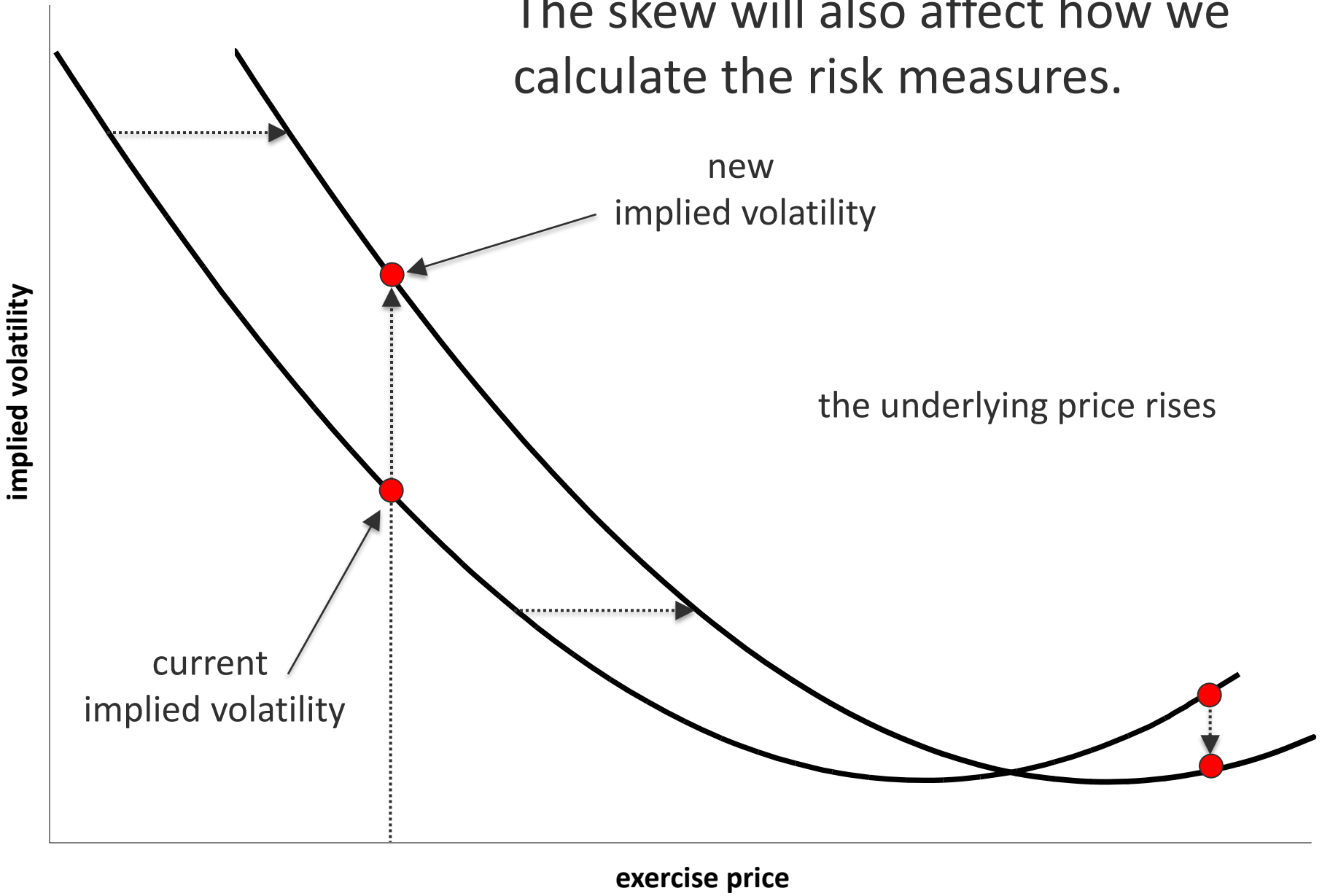
The skew will appear to become more severe.



To adjust for this effect we can change the X-axis scale. Instead of expressing exercise prices in traditional terms, we can express them in terms of standard deviations.



The skew will also affect how we calculate the risk measures.



underlying price = 100.00

95 put = 2.00

implied volatility = 26.0% implied delta = -25

underlying price rises to 101

95 put ≈ 2.00 - (.25 x 1.00) = 1.75

shifted implied volatility = 27.0%

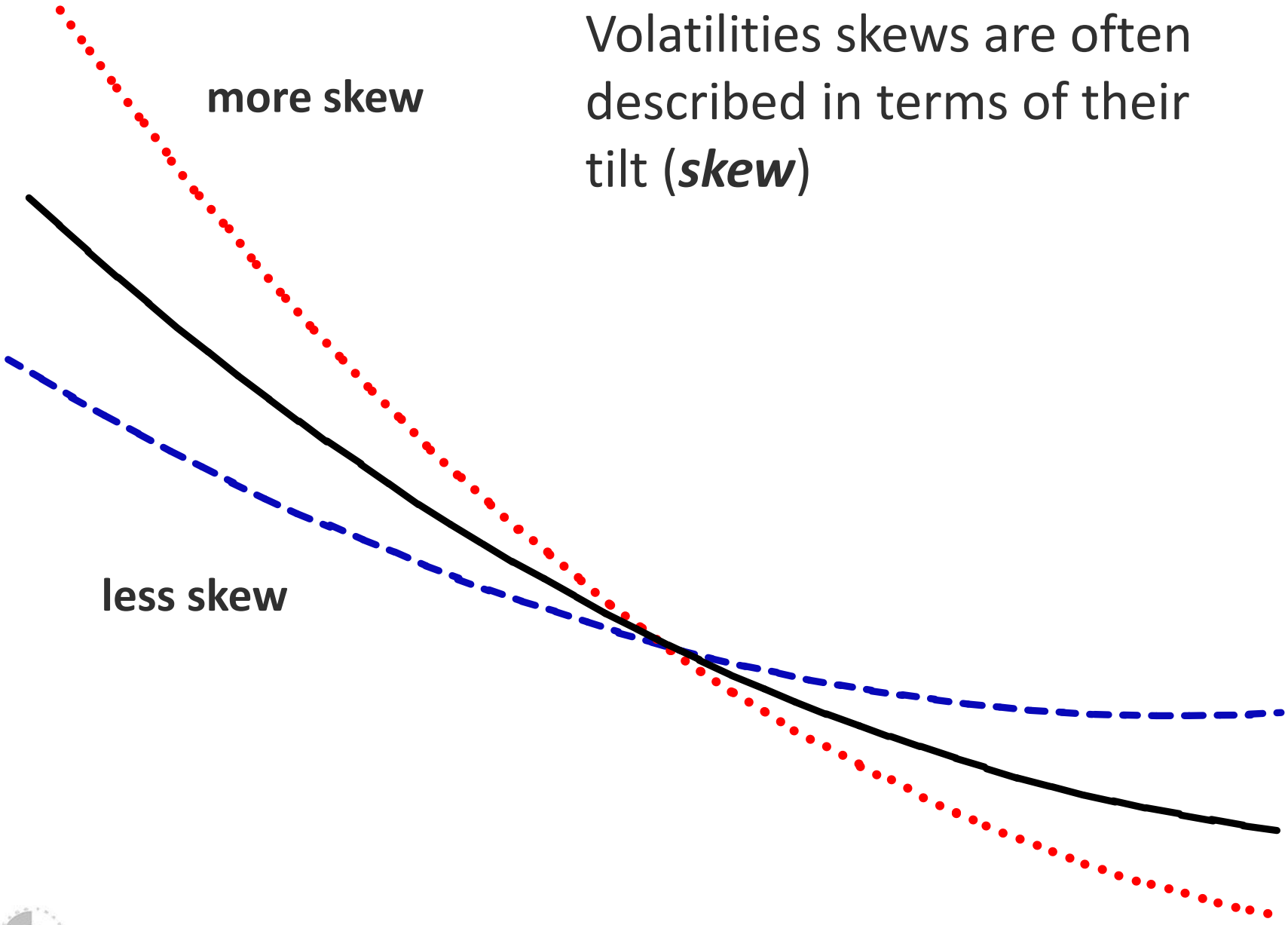
vega of 95 put = .07

95 put ≈ 1.75 + (1 x .07) = 1.82

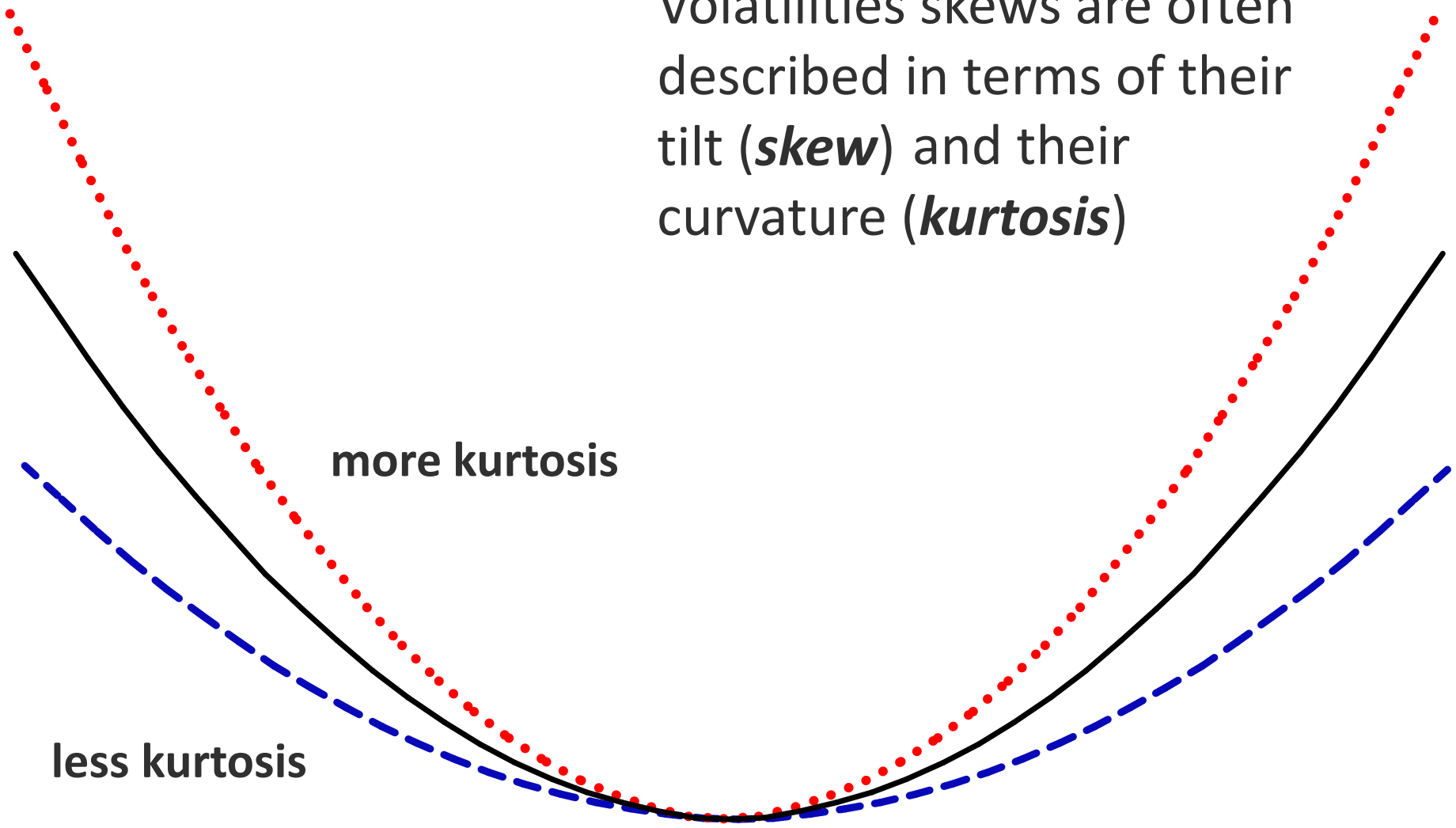
delta of 95 put = (1.82 - 2.00) / (101 - 100) = -.18

adjusted or ***skewed delta = -18***

Volatilities skews are often described in terms of their tilt (***skew***)



Volatilities skews are often described in terms of their tilt (***skew***) and their curvature (***kurtosis***)



Skew Strategy (*Risk Reversal*)

buy (sell) puts at a lower exercise price

sell (buy) calls at a higher exercise price

buy (sell) underlying contracts so that the entire position is delta neutral.

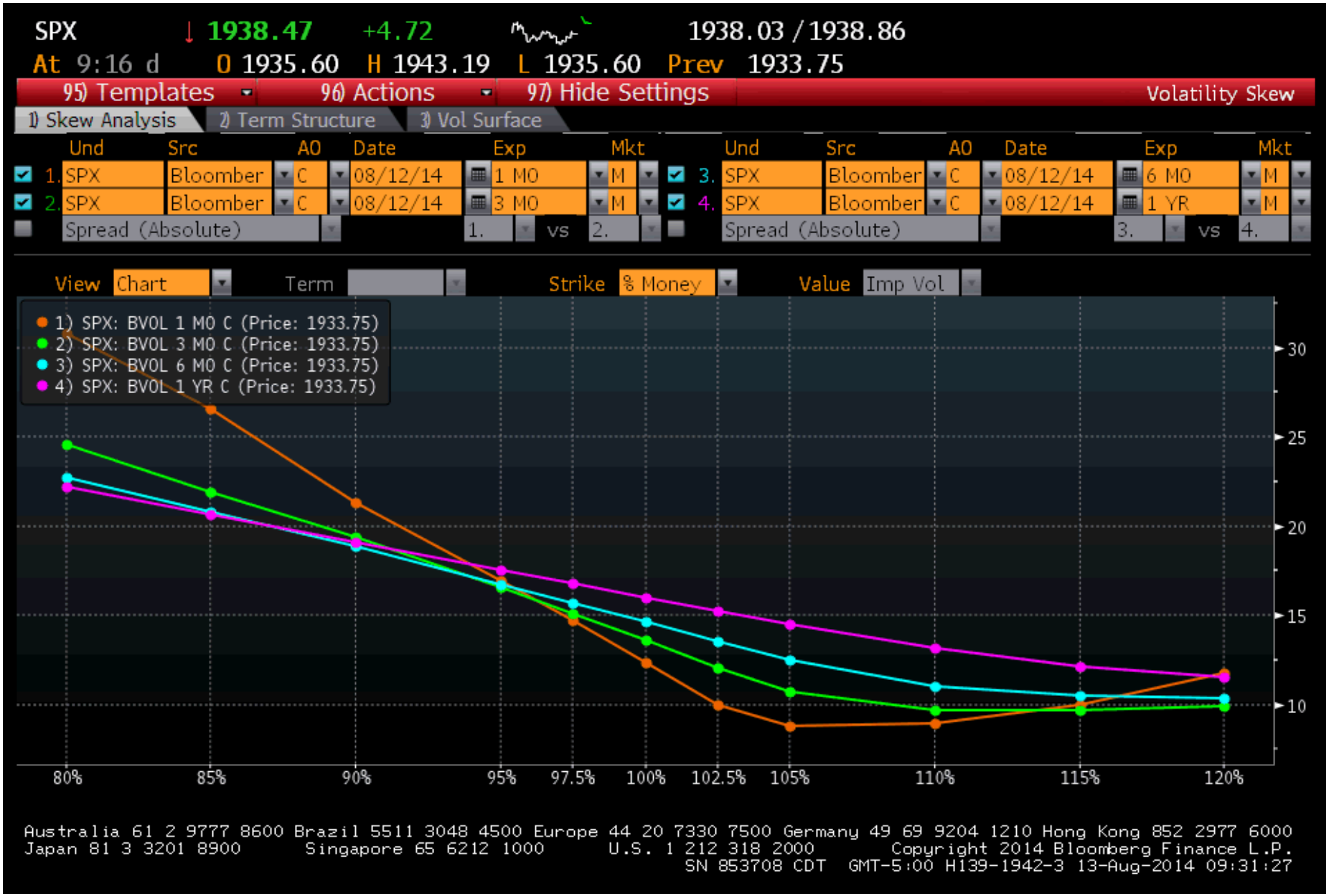
A common measure of volatility skewing is the difference between the implied volatility of the -25 delta put and the implied volatility of the 25 delta call (the 25 delta risk reversal).

Kurtosis Strategy

buy (sell) out-of-the-money strangles

take an opposing position in at-the-money straddles so that the entire position is gamma and / or vega neutral.

Kurtosis options are often those with deltas in the range of 4 to 6 (-4 to -6 for puts).


SPX
1938.47
+4.72
1938.03 / 1938.86
At 9:16 d
O 1935.60
H 1943.19
L 1935.60
Prev 1933.75
95) Templates
96) Actions
97) Hide Settings
Volatility Skew
1) Skew Analysis
2) Term Structure
3) Vol Surface
Und Src AO Date Exp Mkt
1. SPX Bloomber C 08/12/14 1 MO M
2. SPX Bloomber C 08/12/14 3 MO M
Spread (Absolute) 1. vs 2.
3. SPX Bloomber C 08/12/14 6 MO M
4. SPX Bloomber C 08/12/14 1 YR M
Spread (Absolute) 3. vs 4.
View Chart
Term
Strike % Money
Value Imp Vol
1) SPX: BVOL 1 MO C (Price: 1933.75)
2) SPX: BVOL 3 MO C (Price: 1933.75)
3) SPX: BVOL 6 MO C (Price: 1933.75)
4) SPX: BVOL 1 YR C (Price: 1933.75)
30
25
20
15
10
80%
85%
90%
95%
97.5%
100%
102.5%
105%
110%
115%
120%
Australia 61 2 9777 8600 Brazil 5511 3048 4500 Europe 44 20 7330 7500 Germany 49 69 9204 1210 Hong Kong 852 2977 6000
Japan 81 3 3201 8900 Singapore 65 6212 1000 U.S. 1 212 318 2000 Copyright 2014 Bloomberg Finance L.P.
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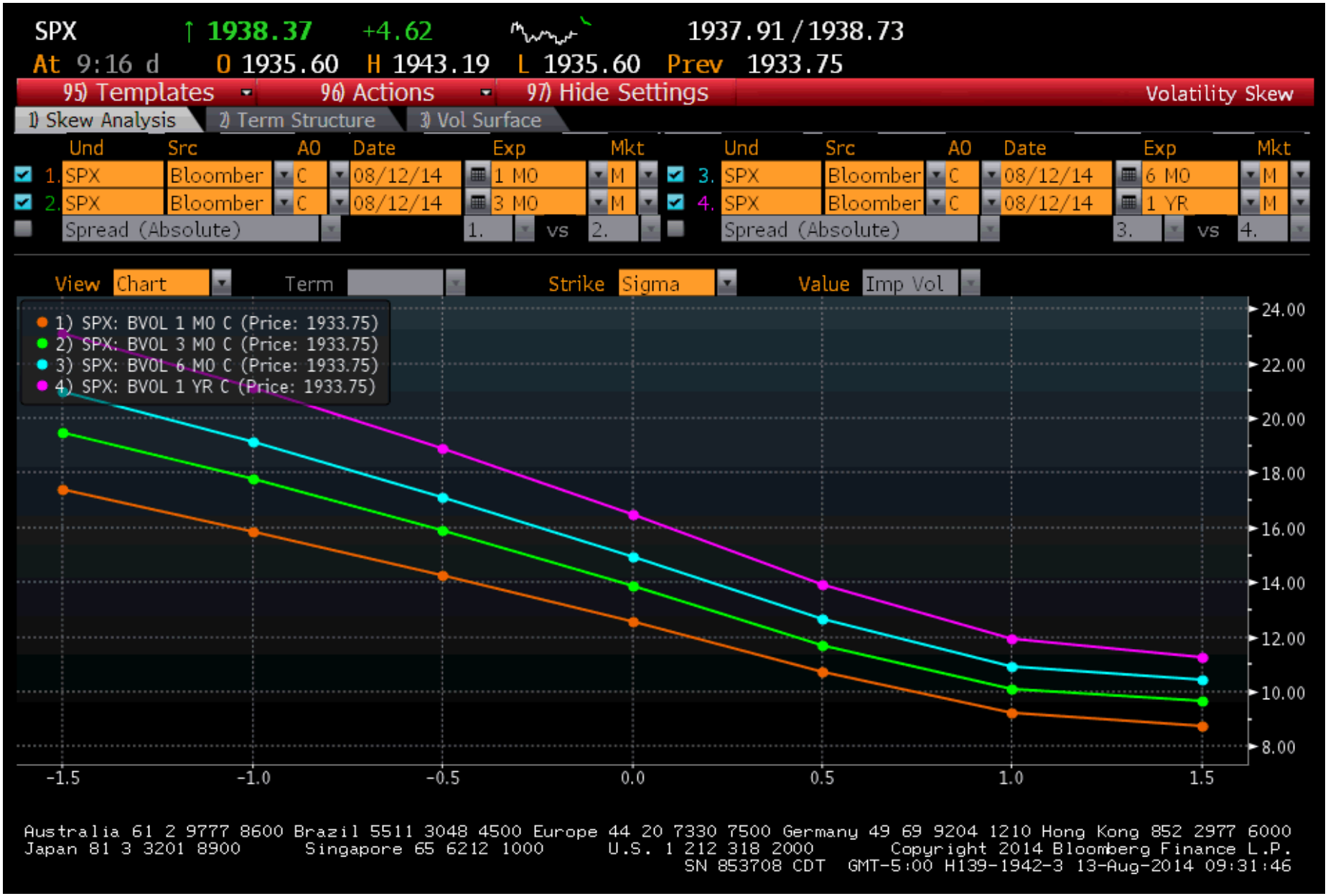
SPX ↑ 1938.37 +4.62 1937.91 / 1938.73
At 9:16 d O 1935.60 H 1943.19 L 1935.60 Prev 1933.75
95) Templates 96) Actions 97) Hide Settings Volatility Skew
1) Skew Analysis 2) Term Structure 3) Vol Surface
Und Src AO Date Exp Mkt
1. SPX Bloomber C 08/12/14 1 MO M
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Spread (Absolute) 1. vs 2.
3. SPX Bloomber C 08/12/14 6 MO M
4. SPX Bloomber C 08/12/14 1 YR M
Spread (Absolute) 3. vs 4.
View Chart Term Strike Sigma Value Imp Vol
1) SPX: BVOL 1 MO C (Price: 1933.75)
2) SPX: BVOL 3 MO C (Price: 1933.75)
3) SPX: BVOL 6 MO C (Price: 1933.75)
4) SPX: BVOL 1 YR C (Price: 1933.75)
24.00
22.00
20.00
18.00
16.00
14.00
12.00
10.00
8.00
-1.5
-1.0
-0.5
0.0
0.5
1.0
1.5
Australia 61 2 9777 8600 Brazil 5511 3048 4500 Europe 44 20 7330 7500 Germany 49 69 9204 1210 Hong Kong 852 2977 6000
Japan 81 3 3201 8900 Singapore 65 6212 1000 U.S. 1 212 318 2000 Copyright 2014 Bloomberg Finance L.P.
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